Turbulent convection

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I. INTRODUCTION

Turbulent convection in a fluid heated from below (Rayleigh-Bénard convection or RBC; for recent more detailed reviews, see [1, 2]) plays a major role in numerous natural and industrial processes. It occurs in Earth's outer core [3, 4], atmosphere [5, 6], and oceans [7, 8], and is found in the outer layer of the sun [9] and in giant planets [10]. A beautiful example is seen in the photosphere of the Sun (see Fig. 1), where a dominant feature is an irregular polygonal pattern of bright areas surrounded by darker boundaries. These granules are convection cells with a width of typically 10^3 km.

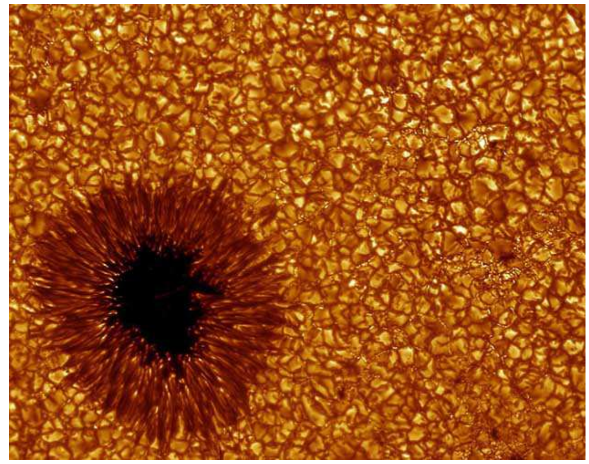

FIG. 1: Granules and a sunspot in the Sun's photosphere, observed on August 8, 2003 by Gran Scharmer and Kai Langhans with the Swedish 1-m Solar Telescope operated by the Royal Swedish Academy of Sciences.

The processes mentioned in the previous paragraph are exceptionally complex. It is true that buoyancy due to the density variation associated with the temperature variation and in the presence of gravity is the central driving force that produces the fluid flow. However, this flow often is modified by the influence of a Coriolis force, for instance due to the rotation of an astrophysical object. Further complications arise from the fact that the fluids involved sometimes are plasmas or liquid metals. In those cases the flow can interact with or even generate magnetic fields. The equations of fluid mechanics, i.e. the Navier-Stokes (NS) equations, then have to be supplemented by and are coupled to Maxwell's equations. Additional problems may be added by the shape of the convecting system which can introduce complicated boundary conditions. What then is a physicist to do in this situations of apparently hopeless complexity? The astrophysicist or engineer, for instance, will have to come to grips with the entire problem by making whatever approximations may be necessary to render it tractable while not loosing any of the main physical aspects. The physicist, on the other hand, has the luxury of extracting a particular manageable aspect from the whole and idealizing it in a carefully constructed laboratory apparatus or computer program where boundary conditions and other external conditions are precisely defined. In this idealized system quantitative studies of particular fundamental aspects of the complex system then become feasible.

II. CONVECTION IN A CYLINDER

The idealization to be discussed further in this article is a sample of fluid in a cylindrical container with a circular cross section, a vertical axis, an aspect ratio $\Gamma \equiv D/L$ (D is the diameter and L the height), and heated uniformly over its bottom surface while it is cooled uniformly from above. In addition to its relevance (or some may say irrelevance because of the major idealization) to astrophysics and geophysics (as well as numerous industrial applications), this system turns out to be of remarkable interest for its own sake. From the fluid mechanics viewpoint, it is fascinating because it is dominated over wide parameter ranges by the physics of boundary layers. Equally interesting is that it provides a tractable example of interactions between large and small scales which are broadly important in fluid-flow problems. More generally from the viewpoint of statistical mechanics, it offers the opportunity to study the statistical properties of a driven (i.e. non-equilibrium) system in which the small turbulent scales are the noise source that drives the large-scale flow structures.

III. WHAT DO WE KNOW ABOUT THIS SYSTEM?

A. Below the onset of turbulence

It is convenient for comparison between different systems to express the strength of the thermal driving in terms of a dimensionless form of the temperature difference

$$Ra = \frac{\alpha g \Delta T L^3}{\kappa \nu} \tag{1}$$

known as the Rayleigh number. Here α is the isobaric thermal expansion coefficient, g the local acceleration of

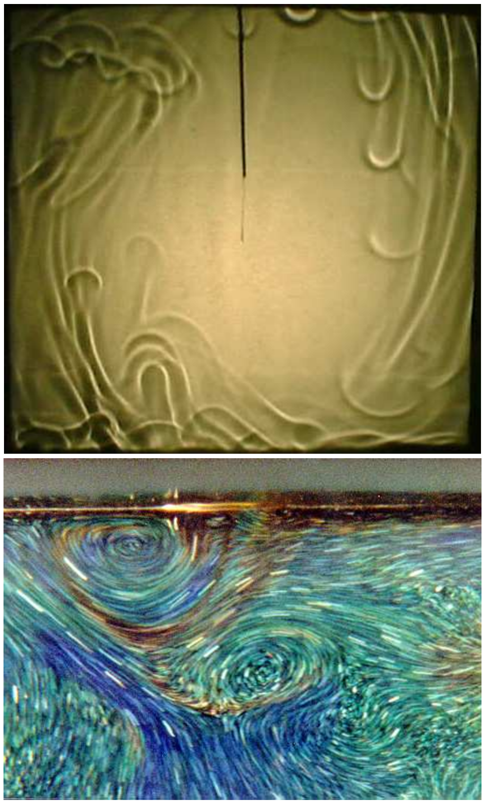

FIG. 2: Upper figure: Shadowgraph visualization of rising and falling plumes at $Ra = 6.8 \cdot 10^8$, $Pr = 596$ (dipropylene glycol) in a $\Gamma = 1$ cell. From [11]. Lower figure: Streak picture of temperature sensitive liquid crystal spheres taken near the top surface in a $\Gamma = 1$ cell at $Ra = 2.6 \cdot 10^9$ and $Pr = 5.4$ (water), in order to visualize plume detachment. The view shows an area of 6.5cm by 4cm. From [12].

gravity, ΔT the applied temperature difference, κ the thermal diffusivity, and $\nu = \eta/\rho$ the kinematic viscosity (η is the shear viscosity and ρ the density). For sufficiently small ΔT the motionless (pure conduction) state of the fluid is stable, and convection will set in only when $Ra > Ra_c(\Gamma)$. It has long been known that $Ra_c(\infty) = 1708$, but for a cylinder of finite Γ Ra_c is larger and depends on the conductivity of the side walls.[13] Here we consider the case $\Gamma = 1$. For non-conducting walls one then has $Ra_c(1) \simeq 3700$. Above onset the azimuthal symmetry of the fluid flow can be described well by the eigenfunctions of the Laplace operator in cylindrical coordinates, i.e. it has the form $\exp(i\ m\ \theta)$. For our $\Gamma = 1$ case and close to Ra_c the flow consist of a single convection roll with up-flow along the wall at an azimuthal orientation θ_0 and down-flow at $\theta_0 + \pi$, corresponding to $m = 1$. As Ra increases, the pattern becomes more complex, corresponding to larger values of m. When Ra is sufficiently large, time dependence sets in. Precisely what happens will depend on the Prandtl number $Pr \equiv \nu/\kappa$ (which tells us about the relative importance of viscous and thermal dissipation). Typically, the time dependence at first is chaotic, remnants of the cellular flow structure with $m > 1$ are still recognizable, and the fluid flow remains laminar; but as Ra is increased further beyond some Ra_t, all internal structure disappears except for a single roll ($m = 1$). In that Ra range vigorous small-scale fluctuations become important, and we regard the sample as being turbulent. The value of Ra_t depends both on Γ and on Pr[14]. For $Pr \simeq 30$ and $\Gamma = 1$ for instance we found $Ra_t \simeq 10^7$.

B. The turbulent range

For $\Gamma \simeq 1$ and $Ra > Ra_t$ much experimental and numerical work was done (for details see [1]). It showed that this system once more contains a single convection roll, known as the "Large-Scale Circulation" or LSC, just as it did close to Ra_c, albeit in the presence of vigorous fluctuations on smaller length scales. The upper part of Fig. 2 is a shadowgraph visualization, looking sideways through the sample. This method is based on the bending of light rays by refractive-index gradients and thus provides an image closely related to the temperature field. One sees plumes of relatively warm fluid rising on the left and ones of relatively cold fluid falling on the right. These plumes originate at thermal boundary layers (BLs) [15] of thickness $\lambda_b << L$ just below the top and just above the bottom plate. An example of plume emission is shown in the bottom of Fig. 2. As a very crude approximation, the BLs can be viewed as quiescent fluid, with each of them supporting a temperature difference roughly equal to $\Delta T/2$. This then would leave the entire sample interior at a nearly constant temperature. In reality the situation is a great deal more complicated because the temperature and velocity fields are fluctuating vigorously, both in the interior and in much of the BLs. Again roughly speaking, the BLs will adjust their thickness so that, according to Eq. 1, the Rayleigh number based on λ_b (rather than L) approximately reaches its critical value. The plume emission can then be viewed as a manifestation of the near-marginal stability of the BLs.

Recent experimental work for $\Gamma \simeq 1$ revealed that the LSC, carrying the plumes and in turn being driven by their buoyancy, displays most interesting dynamics. In samples of circular cross section the orientation of the near-vertical circulation plane undergoes azimuthal diffusion, as revealed by the observation that its mean square azimuthal displacement is proportional to the elapsed time.[16–19] A further fascinating feature of the LSC is a torsional oscillation, with azimuthal displacements that are out-of-phase by π in the top and bottom parts of the sample.[20, 21]. An important question was whether this mode is a characteristic of an underlying dynamical system, having come into existence perhaps via a Hopf bifurcation hidden away somewhere in parameter space. Such a deterministic oscillator mode would have a probability distribution $p(\theta - \theta_0)$ of the azimuthal displacement θ away from the mean value θ_0 with two maxima, one each at the two displacement extrema. However, it turned out that $p(\theta - \theta_0)$ was Gaussian distributed,[21] with a maximum at $\theta - \theta_0 = 0$. Such a distribution is indicative of a stochastically driven damped harmonic oscillator.[22] Thus both the azimuthal diffusion and the nature of the torsional mode suggest to us that we are dealing with a large-scale structure (i.e. the LSC) of the system that is driven by the noise produced by the small-scale turbulent background fluctuations.

Another experimentally observed property of the LSC is that its circulation occasionally slows down and virtually comes to a halt, only to start up again, albeit usually at a different orientation.[18, 23] These "cessations" are events reminiscent of the cessations observed in the geo-dynamo that are associated with reversals of Earth's magnetic field.[3, 4] Much earlier it had been realized already that there are also rare occasions when the LSC orientation undergoes rotations at exceptionally high rates without completely loosing its circulation.[24] Both the "rotations" and the cessations occupy only a small fraction of the total time, and are superimposed upon the otherwise diffusive azimuthal dynamics.

Yet another unexpected experimental observation was that the probability distribution of θ_0 had a broad peak rather than being uniform as would be expected based on the rotational invariance of the system.

Stimulated by some of these experimental findings and hopeful for an explanation of others, Eric Brown and I derived a simple model for the LSC.[25, 26] The idea was to identify the smallest number of necessary components of the LSC, to retain the terms of the NS equations that are physically relevant to these components, to perform a volume average so as to reduce the field equations to ordinary differential equations, and to add phenomenological stochastic driving terms (with intensities derived from the measured diffusivities) to represent the action of the small-scale fluctuations on the large-scale excitation. There turn out to be at least two necessary components, namely the circulation strength U and the azimuthal orientation θ_0 of the circulation plane. The strength U is driven by the buoyancy term and damped by viscous velocity boundary layers near the walls. The equation for U is coupled to that for θ_0 by a term that arises from the nonlinear term in the NS equation; this term represents the angular momentum of the LSC and is proportional to U. We assume further that U is proportional to the amplitude δ of the measurable sinusoidal temperature variation around the circumference at the horizontal mid-plane of the cylinder. This procedure yielded the two equations

$$\dot{\delta} = \frac{\delta}{\tau_\delta} - \frac{\delta^{3/2}}{\tau_\delta \sqrt{\delta_0}} + f_\delta(t) \ , \qquad \ddot{\theta}_0 = -\frac{\dot{\theta}_0 \delta}{\tau_{\dot{\theta}} \delta_0} + f_{\dot{\theta}}(t) \quad (2)$$

with the characteristic times τ_δ and $\tau_{\dot{\theta}}$ and the mean value δ_0 of δ determined by other measurable parameters of the system [1, 25, 26].

The deterministic part of the equation for $\dot{\delta}$ can be written as $\dot{\delta} = -\partial F/\partial \delta$ with the potential $F = -\delta^2/(2\tau_\delta) + 2\delta^{5/2}/(5\tau_\delta\sqrt{\delta_0})$. One sees that there is an unstable fixed point at $\delta = 0$ and a stable one at $\delta = \delta_0$. Normally δ will undergo diffusion in the depth of the potential well surrounding δ_0; but on rare occasions δ will be driven by the noise to the unstable fixed point. Such an event correspond to a cessation. The equation for $\ddot{\theta}_0$ is equally interesting. Reflecting the rotational invariance of the system, it has no potential extrema. It will yield diffusion, but at a typical rate controlled by the effective damping term proportional to δ on the rhs. As mentioned, this term represents the angular momentum of the LSC. Immediatly one sees that rapid and large changes of θ_0 can, but do not have to, occur when δ (and thus the angular momentum) is small. This feature explains the observed occasional rapid rotations.

Recently the model was extended by including terms that break the azimuthal invariance of the system.[27]. An example of such a term is a non-circular cross section of the cylinder. The model then predicts that the circulation plane will tend to align along the largest diameter, with fluctuations about this alignment. Another example is a system with a tilt of the vertical axis relative to gravity. Both of these cases will, for appropriate parameter values, lead to oscillations of θ_0 corresponding to a damped stochastically driven harmonic oscillator. For the tilted case these oscillations have actually been observed and their properties have been measured.[27] Note that these oscillations are unrelated to the torsional oscillations mentioned earlier.

A particularly interesting symmetry-breaking term is Earth's Coriolis force which couples to the circulation.[17] In the northern hemisphere it turns out that up- or down-flow, more or less parallel to the cylinder axis, yields a preferred westerly orientation of θ_0, whereas flow more or less horizontal and thus parallel to the cylinder diameter applies a torque that tends to rotate the circulation plane in the clockwise direction when seen from above. These two competing effects yield a periodically varying potential (with period 2π) with a sloping background. Such a potential is sometimes known as a "washboard potential" and arises in many condensed-matter physics problems, including charge-density waves in semi-conductors and constant-current-biased Josephson junctions. Knowing the azimuthal diffusivity and the potential of the system, one can calculate the probability distribution $p(\theta_0)$ using a Fokker-Planck equation. The result, obtained without any adjustable parameters, agrees extremely well with the measured broad peak in $p(\theta_0)$ that had been so surprising in view of the perceived rotational invariance of the system. Here we have a wonderful application of the methods of statistical mechanics to a fluid-mechanical problem.

Extensive measurements were made also for cylinders with $\Gamma = 0.5$ and $Pr = 5$.[16, 28, 29] Among other interesting results, this work showed that cessations are more frequent by an order of magnitude than they are for $\Gamma = 1$. It remains to be seen whether this difference can be explained in terms of the model represented by

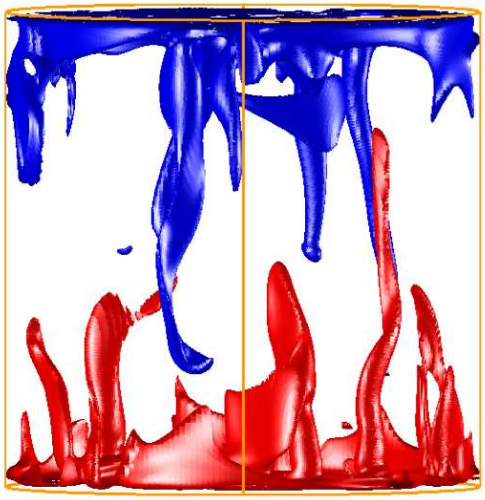

FIG. 3: Visualization for $Ra = 10^8$ of two temperature isosurfaces in a cylindrical sample with $\Gamma = 1$ for $Pr = 6.4$ and at a modest rotation rate. From [30].

Eqs. 2 with appropriate parameter choices .

IV. WHAT ARE THE UNRESOLVED ISSUES?

There is a number of variations of the basic RBC problem that are of current interest. One of them is the influence of deliberately imposed rotation about an axis parallel to the cylinder axis and at angular speeds Ω much larger than that of Earth's rotation. For Ω not too large the Coriolis force will twist the plumes emitted from the BLs into vertically aligned tubes known as Ekman vortices. This is illustrated by the direct numerical simulation (DNS) results shown in Fig. 3. These vortices, by virtue of the reduced pressure along their axes, will extract extra fluid from the BLs and significantly increase the heat transport. Nusselt-number enhancements by over 30% have been observed both in experiment and in DNS.[30] However, at larger Ω the Nusselt number is suppressed because globally the rotation suppresses flow parallel to the rotation axis. This is explained by a result from fluid mechanics known as the Taylor-Proudman theorem. Understanding these phenomena has significant industrial consequences, for instance in the growth of crystals from the melt. It is relevant as well to the elucidation of convection in astrophysical objects where rotation can have a much larger influence than it does on Earth. Much more is to be learned about the physics that is involved.

Another interesting problem arises when the applied temperature difference straddles a first-order phase transition.[31] The heat transport can then be enhanced by an order of magnitude or more. This problem is important for instance in understanding the formation of rain in clouds and for the understanding of convection in Earth's mantle[32]. And of course it has numerous industrial applications.

Returning to pure RBC without the above variations or complications, there also remain major open issues. Let us consider just two of them. First, it is obvious that convection in a cylinder with $\Gamma \simeq 1$ does not correspond very closely to many of the problems of interest, for instance to the granules seen in the photosphere of the Sun (see Fig. 1). We would love to know whether an irregular polygonal pattern of convection cells such as seen in Fig. 1 would be the plan-form of the LSC in a system of very large Γ. To answer this simple question is difficult. In experiments there generally is a limit to the lateral extent of an apparatus. Thus large Γ often is achieved only at the expense of the height L. However, small L according to Eq. 1 will lead to small Ra, and yet large Ra is desired as well. Nonetheless this no doubt will be one of the directions of future research. If an irregular polygonal pattern does indeed exist, then an interesting question will be how this pattern is influenced by a prevailing lateral current imposed upon the system. This issue is relevant for instance to the formation of cloud streets (lines of cumulus clouds) in the atmosphere. It has been studied at some length near the onset of convection,[33] but to my knowledge not for the turbulent system. The common view is that the irregular convection cells will be organized into more or less ordered rolls by the prevailing wind.

A second question of great importance is how RBC, even in a cylinder with $\Gamma = \mathcal{O}(1)$, will behave at very large Ra. With a few exceptions to be mentioned below, laboratory experiments have been limited to $Ra \lesssim 10^{12}$. DNS have not yet been able to reach such high values. Reliable calculations, taking many days of CPU time on modern computers, have reached $Ra \simeq 10^{10}$ for a cylindrical sample with $\Gamma = 1/2$.[34] In the explored Ra range measurement and DNS indicate that $Nu \propto Ra^{\gamma_{eff}}$ with γ_{eff} changing gradually from about 0.28 to about 0.31 as Ra changes from 10^7 to 10^{12}.[35–38] This behavior is explained very well by a model of Grossmann and Lohse[39, 40] which is based on a decomposition of the kinetic and thermal dissipations into boundary and bulk contributions. As Ra increases, bulk contributions generally become more important and for that reason the effective exponent changes.

One might be quite satisfied with the understanding of RBC developed on the basis of the existing measurements for $Ra \lesssim 10^{12}$, except for the fact that theoretically the physics of this system is expected to change dramatically as Ra grows further. With increasing Ra the LSC is expected to become more vigorous. Its maximum speed is near the BLs at the top and bottom; but directly at the plates the velocity has to vanish for a viscous fluid. Thus the LSC applies a shear to the BLs. When the shear becomes large enough, the heretofore laminar (albeit fluctuating) BLs will themselves become turbulent and in a sense be swept away. An estimate [41] suggests that this will occur for $Ra = Ra^* \simeq 3 \times 10^{14}$ when

$Pr = 1$, and that $Ra^* \propto Pr^{0.7}$. The nature of $Nu(Ra)$ for $Ra > Ra^*$ was investigated theoretically long ago by Kraichnan [42], and his predictions have stimulated the cummunity ever since to search for ways to explore this high-Ra regime. His prediction for a system without BLs is that $Nu \propto Ra^{1/2}$, i.e. that Nu should increase much more rapidly with Ra than it does below Ra^*. Of course our actual laboratory system does have top and bottom boundaries, and even though the laminar BLs may be gone, there remains the restriction that the velocity must vanish at the solid-liquid interface. This condition leads to so-called "viscous sublayers" which are thinner than the laminar BLs; in Kraichnan's theory they lead to logarithmic corrections to the relation between Ra and Nu, yielding $Nu \propto Ra^{1/2}/[ln(Ra)]^{3/2}$.

There are at least two reasons why the Kraichnan transition is so important. First, it is associated with a fundamental change in the heat transport mechanism. Below Ra^* the heat transport was limited primarily by laminar BLs. Above Ra^* the limiting factor presumably is a thermal gradient in the bulk fluid. Second, we know that what we learned below Ra^* can not be extrapolated to $Ra > Ra^*$ because of this change in the mechanism. It turns out that many of the astrophysical applications involve $Ra \gtrsim 10^{20}$, i.e. values above Ra^* by several orders of magnitude. So we really can not extrapolate existing measurements to the Ra-ranges of these natural phenomena.

How then can we reach very large values of Ra? From Eq. 1 one sees that either a fluid can be chosen for which the combination $\alpha/\kappa\nu$ is particularly large, or an apparatus with very large L can be build. The former choice was pursued by a group in Grenoble, France who used fluid helium at about 5K near its critical point and reached $Ra \simeq 10^{15}$.[43] Another group, in Oregon, went further by using helium as well, and by at the same time also constructing a large apparatus with $D \simeq 0.5$m and $L \simeq 1$m.[44] Unfortunately the two sets of measurements do not agree. The Grenoble results found a transition in $Nu(Ra)$ at $Ra \simeq 10^{11}$ from a low-Ra regime with $\gamma_{eff} \simeq 0.31$ to a high-Ra regime with $\gamma_{eff} \simeq 0.39$ which they interpreted as the Kraichnan transition even though it occurred at an unexpectedly low value of Ra^*. The Oregon group reached unprecedented values of Ra as large as 10^{17}; their data were consistent with $\gamma_{eff} \simeq 0.31$ over their entire Ra range and did not reveal any transition. It is interesting to note that $Nu \simeq 20000$ when $Ra \simeq 10^{17}$!

It seemed desirable to address the above conflict by an experiment that was not dependent on cryogenic techniques and that used classical fluids at ambient temperatures. To that end Denis Funfschilling, Eberhard Bodenschatz, and I used a very large pressure vessel at the Max Planck Institute for Dynamics and Self-Organization in Göttingen, Germany. It is a cylinder of diameter 2.5 m and length 5.5 m, with its axis horizontal, and with a turret above it that extends the height to 4 m over a diameter of 1.5 m. Because of its suggestive shape, this vessel has become known as the "Uboot of Göttingen". It can be filled with various gases at pressures up to 19 bars. In the section containing the turret we placed a RBC sample-cell with $L = 2.24$ m and $D = 1.12$ m (the "High Pressure Convection Facility" or HPCF), yielding $\Gamma = 0.500$.[45] Figure 4 shows the insertion of the HPCF into the turret of the Uboot. After insertion, a dome is placed on top of and bolted to the turret section to complete the pressure enclosure. Using sulfur hexafluoride at 19 bars as the gas, we reached $Ra \simeq 10^{15}$. Consistent with the Oregon experiment, but differing from the Grenoble measurements, we did not find the Kraichnan transition in $Nu(Ra)$. Work with the HPCF is still under way, and we look forward to what the future will bring. We hope to learn quite a bit more about how the LSC evolves as Ra becomes large. However, at this point it is not clear whether the ultimate, or asymptotic, regime predicted by Kraichnan can ever be reached in a system with rigid top and bottom plates. But then the granules in the Sun's photosphere for instance do not have any such confining plates!

FIG. 4: The High-Pressure Convection Facility, weighing approximately 2000 kg, is being inserted into the turret of the Uboot.

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