Over Two Decades of Pattern Formation, a Personal Perspective


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Abstract: Patterns are ubiquitous in the world that surrounds us. They can form via bifurcations, for instance from the spatially uniform state, as a control parameter is varied. Their nature generally is determined by nonlinear terms in the relevant equations of motion, and thus their elucidation is a non-trivial goal in nonlinear physics. In the early 1970's, there was a revival of interest in the condensed-matter physics community in chaos and pattern formation in nonlinear dissipative systems. Experimentalists and theorists brought the tools of their field to bear on these challenging problems. Mostly in terms of his own experiences, the author of this paper reviews some of the issues that have been addressed, some of the techniques that have been applied, and some of the progress that has been made by experimentalists during the two-and-a-half decades since then, and the relationship which these results have to our present-day understanding of nonlinear systems.

1 Introduction

When a system is removed far from equilibrium by subjecting it to a stress, it will often undergo a transition from a spatially uniform state to a state with spatial variation. We refer to this variation as a "pattern". Pattern formation is generally associated with nonlinear effects. These are of great fundamental interest to physicists because they often lead to qualitatively new phenomena which do not exist in linear systems and which are not yet fully understood.
Fig. 1. Three systems in which rolls (also called stripes) or hexagons occur over different parameter ranges. a): Chemical patterns obtained by Ouyang and Swinney. [1] b): Patterns in a gas discharge system obtained by Breazeal, Flynn, and Gwinn. [2] c): Patterns in Rayleigh-Bénard convection obtained by Bodenschatz et al. [3] (hexagons) and by Hu et al. [4] (rolls).

Another fascinating aspect of pattern-formation phenomena is that many of them have a universal character. Even though the system under investigation may for instance be of physical, chemical, or biological nature, many common features are encountered. Pattern formation has been studied in such diverse fields as fluid mechanics, optics, chemistry, biology, solid-state physics, gas discharges, and others. Figure 1 illustrates some of these. In Fig. 1a chemical patterns obtained by Ouyang and Swinney [1] are illustrated. Two different mixtures of reactants are circulated through two channels across the outside of porous plates.
Between these plates, a layer of a gel permits mixing by diffusion of the chemicals, which leads to a spatially dependent chemical reaction. A spatial variation develops in the concentration fields. Among other patterns, hexagons and "stripes" or "rolls" are obtained, depending on the parameter ranges. In Fig. 1b we see patterns, obtained by Breazeal, Flynn, and Gwinn, [2] of visible light emitted from a gas subjected to a large alternating electric field. Here also hexagons and stripes are observed over a range of parameter values. Finally, in Fig. 1c, we see the result of Rayleigh-Bénard convection. A fluid is contained between two parallel horizontal plates separated by a distance $d$ and heated from below by a heat current $Q$. When the fluid properties do not vary significantly over the applied temperature difference, convection rolls are obtained. [4] These are similar to the stripes in Figs. 1a and 1b. When there is a significant variation of the fluid properties, hexagons occur. [3, 5] The communality (or universality) between these diverse systems is of course provided by the common features of the mathematical equations which describe these systems. A scientist interested in the fundamental aspects of these ubiquitous effects will choose a particular system which is well suited to detailed, quantitative study. The universal aspects of patterns provoke hope of a deeper understanding in very complicated systems (where the equations of motion might not even be known) from the study of simpler ones. For instance one might expect to learn something about the formation of biological patterns which are difficult to study under highly controlled conditions from work on simpler physical or chemical systems where boundary conditions and control parameters can be specified precisely in an experiment.

Aside from the intellectually fascinating aspects of pattern formation, there are practical reasons for being interested in this field. Our physical surroundings are dominated by patterns. We see ripples on a pond, regular or irregular cloud formations, the remarkable shapes of snowflakes, and the patterns in animal skins or coats. The surprising aspect is the great regularity which is often encountered. A deeper understanding of these natural phenomena will enrich our lives and deepen our appreciation of our environment. There are also more practical benefits for our society which will arise from a more fundamental understanding of patterns. Their better understanding enables us to prevent undesirable or promote advantageous patterns in industrial processes. Examples include the displacement of oil by water in a porous oil reservoir, in which finger-like patterns formed by the water can reduce the yield. Others can be found in crystal-growth processes, where patterns in the melt or the solid phase may be undesirable. Patterns occur in the wake of airplane wings, in chemical reaction fronts, flames, leaching of minerals from ore beds, fronts between infected and uninfected portions of a population, and in many other technologically, economically, biologically, or sociologically important situations. In the long run their fundamental understanding clearly will have far-reaching consequences for our society. An extensive review of the physics of pattern formation in physical, chemical, and biological systems was published recently by Cross and Hohenberg. [6]

I have been very grateful for my serendipitous introduction to the field of nonlinear nonequilibrium systems, which eventually led to my interest in the
experimental study of patterns. In 1970, I was a staff member at Bell Laboratories. My interests at that time were primarily in the field of critical phenomena, with an emphasis on the superfluid transition in liquid $^4$He. Late in that year, my department head Paul Fleury had a visitor from Belgium by the name of Jean Pierre Boon. Jean Pierre told Paul, me, and some of our colleagues about Rayleigh-Bénard convection (RBC). [7] It seems difficult to imagine from our present vantage point; but to my knowledge none of us had ever heard about this phenomenon as an interesting physical system even though of course all of us were familiar with convection from everyday experiences. Today RBC has evolved into one of the primary model systems for the experimental study of pattern formation. Another serendipitous aspect of Jean Pierre’s visit was that I had an apparatus in my laboratory at that time which I used for the study of the thermal conductivity of liquid $^4$He near the superfluid transition at 2.172 K. [8,9] At that temperature, helium has a negative expansion coefficient, and thus I was heating the sample from below so as to avoid convection. It was a simple matter to increase the temperature by a fraction of a degree and then to study the interesting effect of convection upon the heat transport in a temperature range where the expansion coefficient is positive. Since my apparatus was cold and operational at the time of Jean Pierre’s visit, I was able to obtain heat-transport data which were a great deal more precise than previous results in the literature [10] within a day or two. Some of these are shown in Fig. 3 below. They revealed a bifurcation at a well defined value $\Delta T_c$ of the temperature difference $\Delta T$. There was a little rounding of the bifurcation which initially provoked interesting speculations about fluctuation effects; but this rounding is now well understood in terms of small imperfections in the sample geometry. The value of $\Delta T_c$ agreed well with the theoretical prediction.

In the remainder of this paper I would like to present some of the events in the study of pattern formation which I witnessed since the visit by Jean Pierre in 1970. It is my hope that I can convey to the reader the excitement of the field by describing some my own experiences. Under no circumstances should this account be seen as a review of the field. Rather, it is a presentation of some of the important events from a highly personal point of view, as I experienced them, and it neglects many major contributions to the field. Having said this, I hope that I do not even have to apologize to those of my friends whose work I am omitting.

2 The 1970’s and Before

Although there is a long history of the study of bifurcations and pattern formation in fluid mechanics by the applied mathematics and engineering community, physicists for the most part had not really appreciated the interesting aspects of this field prior to about 1970. When physicists finally learned about this fascinating area of study, I believe they soon began to play an important role. The
experimentalists did not feel constrained by the practical needs of the engineer and felt free to concentrate on problems which were just complicated enough to be challenging but still simple enough to be amenable to theoretical analysis and quantitative experimental study. Thus, their hallmark became precise experiments on relatively simple systems which lent themselves to a detailed comparison with theory. Before long this led to a comparison between experiments and theoretical or numerical results at the 0.1% level. In earlier days, researchers often felt that they were on the right track when they achieved agreement at the level of perhaps 10%.

So far as I know, there was relatively little quantitative experimental work by physicists on patterns in the 1960’s (a notable exception to this is the seminal work by Donnelly and collaborators on Taylor-vortex flow which started [11−13] a decade earlier). However, I want to mention two important theoretical results of that decade which had a great impact on subsequent experimental activity. The first is the work of Lorenz, [14] through which it became widely appreciated that systems describable by coupled nonlinear ordinary differential equations can exhibit non-periodic time dependence. Investigations of such systems fall into the field of Dynamical Systems, which by now has reached a certain level of maturity. [15] Far less is understood about irregular variations in systems which are extended also in space (i.e. in patterns), and which thus must be described by partial differential equations. In this latter case, there can be non-periodic variation both in space and in time, and this phenomenon frequently is referred to as spatio-temporal chaos (STC). I mention the work of Lorenz and the field of dynamical systems because there has been some as yet unsubstantiated hope that STC can somehow be understood by using what we know about dynamical systems as a starting point. The elucidation of STC is still very much at the forefront of research today, and will no doubt remain so for some time to come. From my point of view, another important theoretical result of the 1960’s was contained in the paper of Schlüter, Lortz, and Busse, [16] which showed that a stable pattern above the onset of convection in a Boussinesq system consists of parallel straight rolls of arbitrary orientation when boundary effects can be neglected. A good approximation to this state can indeed be found in experiments, and is illustrated in Fig. 1c. A second paper which to me was equally important was the one by Busse [17] in 1967, which among other issues presented a systematic perturbation calculation for non-Boussinesq convection [18, 19] in which hexagons become stable at the bifurcation from the conduction state. Such a hexagonal pattern is also shown in Fig. 1c. It was found to be stable over a range of ε which is consistent with the calculation. [3]

The experimental developments of the 1970’s had, I think, a profound impact in the long run on the study of spatio-temporal complexity. Some of us brought the experimental tools of condensed-matter physics to bear on the problems of fluid mechanics. In particular, the precision measurement-techniques which had been developed for the study of critical phenomena were applied to the study of bifurcations, chaos, and STC. I already described how I was drawn into this field; other experimental physicists who entered it included Pierre Bergé and
Monique Dubois, Jerry Gollub, Albert Libchaber, and Harry Swinney. Perhaps another serendipitous occurrence is that all of these people chose problems from fluid mechanics for their initial projects. It turned out that fluid systems lend themselves extremely well to the quantitative investigations which the experimentalists wanted to undertake. In well chosen fluid systems, extremely precise geometries can be specified, boundary conditions can be well controlled, and control parameters can be held extremely constant or varied in very small steps. Later on of course it became desirable to look at other systems; but fluid mechanics offered many advantages initially.

Although the conventional tools of solid-state physics, such as high-resolution thermometry, lock-in amplifiers, light scattering, and others played an important role, I believe that the most important experimental development of the 1970’s was the advent of the computer in the laboratory. It enabled us to carry out projects which we never would have considered in earlier decades, even though by today’s standards these early efforts at automation and data collection may seem primitive. Already early in 1971, we used a home-made data acquisition system to collect time series of a scalar quantity (the temperature of the bottom plate of a convection cell) which contained several thousand values, and used fast-Fourier-transform methods to obtain their power spectra. [20, 21] A little later we progressed to an LSI-11 based system which was interfaced to a PDP-11/45. [22, 23] Similar developments may well have taken place earlier in other laboratories; but for us they revolutionized the kind of projects that could be tackled. Thus, they not only provided us a new tool, but they also gave us a completely new perspective on what types of experiments to do. On some kind of a logarithmic scale, this was as great a step forward as another which I witnessed as a graduate student a decade earlier, when we started to use DC amplifiers and chart recorders, instead of recording individual voltages in our laboratory notebook after obtaining them by looking through a 10-meter telescope at a ballistic galvanometer. Subsequent changes in computer technology have been perhaps as great or greater; but somehow we began to take these for granted because the data-handling capability of computers was growing continuously. By now this has evolved to the point where we routinely program experiments to run unattended for weeks when long time scales are involved, and collect hundreds of Mbytes of data in such a run.

Let me now get to the physics that was being done. Some of the important scientific results of the 1970’s were, I believe, the quantitative measurements by Bergé and Dubois of various components of the velocity field for RBC in pseudo-one-dimensional systems, i.e. in relatively narrow and long cells where the convection rolls line up parallel to the short (y) axis with their wave vector parallel to the x-axis. [24, 25] Examples of their results are shown in Fig. 2. They confirm that the velocity amplitude varies with the mean-field exponent 0.5 as \( \epsilon \equiv (\Delta T - \Delta T_c)/\Delta T_c \) is increased. Similarly, these authors measured the characteristic length \( \xi \) over which the velocity field grows from zero amplitude at a solid boundary to its bulk value in the interior, and the temporal rate \( \sigma \) at which the system responds to changes in the control parameter. They found the mean-field
exponents -0.5 and 1 respectively. This work, as well as some measurements in our laboratory, [26, 27] culminated [28] in a quantitative comparison of the RB system with predictions based on a Ginzburg-Landau (GL) equation [29], and firmly established the analogy between bifurcations in pattern-forming systems on the one hand and critical phenomena in the mean-field limit on the other. Not only the exponents (which of course follow from Landau’s general assumption of analyticity [29] and are not system specific), but also the coefficients which had been calculated specifically for RBC from the Navier-Stokes equation [27, 28], agreed. Similar work was being done with Taylor-vortex flow [30] by Gollub and Freilich, [31 – 33] by Rehberg, [34] and by Pfister and Rehberg. [35] Taylor-vortex flow (TVF) is also a one-dimensional pattern-forming system, but it differs from RBC in narrow, long cells in that the velocity amplitude at the ends does not vanish, but rather grows to a value of order one. This renders the bifurcation imperfect, and was discussed in terms of a GL equation by Graham and Domaradzki. [36] Rehberg extended the comparison with a GL equation to the secondary Hopf bifurcation to azimuthally travelling waves in TVF. The GL equation has played an enormously important role in the study of patterns during subsequent years, and I believe it is the work which I have described which put its applicability to physical systems on a firm experimental foundation.

At Bell Laboratories, I pursued my studies of convection at cryogenic temperatures. Before long, Robert Behringer and then Robert Walden joined me in this effort. [20, 21, 23, 26, 27, 37 – 43] During the very first measurements using liquid helium, the versatility of cryogenic fluids immediately became apparent. In some of the work we used gaseous 4He because an easily applied change in the pressure could alter $\Delta T_c$, and thus the extent to which the Boussinesq approximation was satisfied, by a large amount. [21, 26, 40] The opportunity to change the fluid properties, and in particular the Prandtl number, by approaching the critical point, was also exploited. [38, 39] As an illustration of some of the early work, results for the Nusselt number [26] are shown in Fig. 3. By changing the sample pressure, precision measurements for Rayleigh numbers up to $150R_c$ were made. Related cryogenic work with an emphasis on even larger Rayleigh numbers was done simultaneously by Threlfall [44], but we were not aware of it until it appeared in the literature.
Fig. 2. Laser-Doppler velocimetry measurements of the velocity in a Rayleigh-Bénard cell by Dubois and Bergé. [24, 25] Left top: horizontal velocity component \( v_x \) near the center of the cell as a function of position \( x \) (\( x \) is in the direction of the roll wavevector). Right top: amplitude of the velocity variation near the center of the cell as a function of \( \epsilon = \Delta T/\Delta T_c - 1 \). Left bottom: horizontal velocity component \( v_x \) as a function of position \( x \) near a sidewall. Right bottom: the healing length \( \xi \) over which the pattern grows from zero amplitude at the wall to its bulk amplitude.

Fig. 3. Left: Time average of the Nusselt number in a cell with \( \Gamma = 5.3 \) as a function of \( R/R_c = 1 + \epsilon \). Right: The power spectrum of the Nusselt number for \( \epsilon = 1.4 \). Adapted from Ref. 26.
A great surprise at the time was that convection in cells of circular horizontal cross section and modest radius was time dependent at relatively small values of $\epsilon$. For an aspect ratio $\Gamma$ (radius/height) of about 5, nonperiodic time dependence started near $\epsilon = 1$. Several years later this phenomenon was confirmed by Libchaber and Maurer. The power spectrum of the Nusselt number was broad, with a maximum at zero frequency, and for large frequency it fell off as $f^{-4}$. This is shown in the right portion of Fig. 3, for $\epsilon = 1.4$. Since here we had found an experimental system with chaotic (i.e. broad band) time dependence, there was a great temptation initially to try to find a connection between the observations and the chaotic behavior of the Lorenz model. This model had been investigated in detail by McLaughlin and Martin. Later we realized that the apparently algebraic falloff was surprising because simple models of chaos in deterministic systems with relatively few degrees of freedom, such as the Lorenz model, had a spectrum with an exponential falloff. As already noted in 1972, it seems likely that the onset of time dependence was associated with wavenumber adjustments as a function of $\epsilon$ which caused the system to cross an instability boundary, from our present vantage point most likely the skewed-varicose (SV) instability. The apparently algebraic falloff of the spectrum presumably is then attributable to the presence of a large but finite number of interacting modes in the spatially extended system which turns out to lead to effectively algebraic decay at intermediate $f$; but so far as I know a quantitative explanation of this phenomenon is still lacking. At very high frequencies the spectrum should then still decay exponentially; but there the power may well be so small as to be unmeasurable. Also still unexplained seems to be the fact that the system remains in the chaotic skewed-varicose-unstable regime, instead of reducing its wavenumber so as to enter once more the regime of stable rolls (this latter phenomenon occurs in the one-dimensional case of a narrow rectangular cell where the SV instability leads to the expulsion of a roll pair and a consequent reduction of the wavenumber).

When $\epsilon$ was increased beyond 3.7 with the cells of $\Gamma \approx 5$, the spectrum of the Nusselt number was modified by a shoulder which developed in the frequency range of algebraic decay. This was also observed in the experiments of Libchaber and Maurer. The power due to this excess contribution grew continuously from zero as $\epsilon$ was increased, and thus was consistent with a forward Hopf bifurcation (albeit with a frequency which was broadened by the underlying chaotic state). The shoulder was at the proper frequency to be attributable to the oscillatory instability (OI) of Clever and Busse. This was so even though of course the calculation was based on the assumption of a bifurcation from the time-independent parallel-roll state.

Much additional work at low temperatures on time-dependent flows in cells of modest $\Gamma$ was done after these early observations, and has been reviewed in detail by Behringer. Here I mention only a surprising result obtained in a very large cell ($\Gamma = 57$). Non-periodic time dependence was found even when the threshold had been exceeded by only 10% or so.
It is really only now gradually being understood [4, 53, 54] in terms of sidewall effects which cause roll curvature, focus instabilities, and local compression of rolls in the cell interior beyond the skewed-varicous instability. Of course it was a great disadvantage of the cryogenic samples that the patterns could not be observed. Today we can do at room temperature with flow visualization much of what was done in the 70's with liquid helium, [3, 4, 53 – 56] and the cryogenic work would be justified only for those few experiments which cannot be done readily at ambient temperature. But two decades ago, the high resolution at low temperatures did open up some new vistas.

Another problem which caught my early interest was the effect of time dependent heating on RBC. The experiments were done with a constant temperature at the top of the cell, and with a step, a temporal ramp, or time-periodic modulation, of the heat current. I was fortunate to be able to interest my colleague Pierre Hohenberg in this problem. He and Jack Swift had worked on the effect of external noise on the Rayleigh-Bénard instability, [57] and their collaboration had produced the now well known Swift-Hohenberg equation. Direct observation of the fluctuations below the bifurcation (which are the response of the system to the external noise) was regarded by most physicists to be nearly impossible because the predicted amplitudes [57 – 62] were so small. Interestingly, at the present time, i.e. 20 years later, these amplitudes have just been measured quantitatively in a Rayleigh-Bénard system. [63] I think Pierre’s interest in the ramping experiments was related to the fact that the flow which evolves as $\epsilon$ is swept slowly through zero should evolve from these fluctuations if deterministic imperfections were small enough. In that case the time evolution of the flow could be used to estimate the size of the fluctuations which prevailed when $\epsilon$ was negative. It was difficult to devise a proper model for the experiment because the pattern which evolved was not known. Nonetheless, with the help of Mike Cross and Sam Safran we carried out a rather detailed analysis. [60] I think in the end we became convinced that deterministic imperfections dominated the early experiments, and it seems to me that the main virtue of the early experimental effort and its analysis was to motivate later, more refined measurements in more carefully constructed cells. [64 – 66] Another important consequence was, however, Pierre’s longterm close interest in my results on nonequilibrium systems, which has been of immense value to all of my subsequent work on patterns. Our direct collaboration on temporally modulated flows continued into the next decade, when Manfred Lücke joined our effort while he and Pierre were at the Institute for Theoretical Physics in Santa Barbara. [67 – 69]

A noteworthy event which I should mention is that early in the 1970’s, Albert Libchaber came to visit us at Bell Laboratories, and apparently became convinced that convection in liquid or gaseous helium had much to offer to the experimentalist. But unlike us, he had the wisdom to understand that without flow visualization the global studies of heat transport, particularly in spatially extended systems, would bring only limited results. Thus, he and Maurer developed local temperature probes. [45] However, soon they concentrated on small systems which could accomodate only about one pair of convection rolls. Naively,
I had always thought that spatially extended systems would be much more interesting. The work by Libchaber and Maurer on small systems led to the now famous discovery of period-doubling bifurcations, and together with the theoretical work by Mitch Feigenbaum [70] it really opened up the field of dynamical systems. [71] Exciting as it is, this work does not involve patterns, and I will not discuss it further here. [15]

Finally, I want to say a word or two about conferences which were important to me during the 70’s. In May of 1973, there was a meeting organized by Jim Gunton and Mel Green at Temple University. It dealt primarily with critical phenomena, and emphasized the new ideas based on the mode-coupling and renormalization-group methods. For this meeting, I had come from Jülich, Germany where I was spending the calendar year on a sabbatical leave from Bell Labs. This gave me a chance to show my experimental results for the $\Gamma_\approx 5$ cell to Paul Martin, who had previously heard about them through Pierre Hohenberg. I think that Paul’s genuine interest in my results played an important role in convincing me that the study of STC should be taken seriously. Evidence of my prior rather casual attitude toward it can be found in the fact that, except for a couple of talks at the January 1972 APS meeting in San Francisco, [20, 21] I had not really published my results. Paul’s interest, and his work with McLaughlin, [46] finally convinced me that I had to write up my work as soon as I returned to Bell Labs early in 1974. [37]

The most important meeting for the field as a whole was, I believe, the NATO ARW organized by Tormod Riste in Geilo, Norway in April of 1975. It was one in a series of workshops on critical phenomena, and was entitled Fluctuations, Instabilities, and Phase Transitions. But Tormod had the vision to see how appropriate it was at that time to devote it primarily to the emerging field of instabilities and patterns. In addition to a large number of theoretical papers, much of the early experimental work by Bergé, Goldberg, Gollub, Whitehead, myself, and others was presented, and this was the first time that some of us met each other. In a sense, one could regard this ARW as the beginning of the study of patterns as a coherent subfield of condensed-matter physics. The increasing appreciation for the importance of this field is also reflected in the fact that a major fraction of a Solvay conference [72] was devoted to it in 1979.

3 The 1980’s

Looking back now, it is apparent that the 1980’s brought both qualitative and quantitative advances to the field of pattern formation. Advances in computer technology, and in particular in affordable storage capacity, permitted more and more advanced experiments to be performed. Instead of taking time series of one or a few scalars, we now could actually take two-dimensional images, or time series of one-dimensional images. "Contour plots", which showed the time evolution of a one-dimensional array, became popular for the documentation
of the dynamics of one-dimensional patterns. One of the early visualizations of
two-dimensional convection patterns was done by Gollub and Steinman, [73] and
was based on laser-doppler velocimetry; but that method turned out to be too
tedious and time consuming when the primary interest was the pattern and not
the quantitative amplitudes of the flow. Instead, the shadowgraph technique was
developed into a very sensitive tool capable of visualizing even extremely feeble
flows very near onset. Of course the method had a long history. After having
been employed in the 1950’s by Silveston, [10, 74, 75] it was developed further
for the investigation of well developed patterns by Busse and Whitehead. [76] Its
application in a more sensitive form to the study of patterns very close to onset
is, however, more recent. [77, 78] By now, we can under favorable circumstances
resolve temperature fluctuations a good bit smaller than a part per million of
the critical temperature difference. The nature of the problems under experi-
mental investigation also became more advanced. As I will describe below, more
advanced bifurcation phenomena and one-dimensional patterns were one area
of interest; but the problem of pattern formation in two dimensions was also
beginning to be addressed.

An important development during this decade was the addition of many new
experimental physicists to the field. At the risk of omitting some and in random
order, I mention that Robert Behringer at Duke, Mike Gorman at Houston,
David Andereck at Ohio State, Bob Walden, Cliff Surko, and Paul Kolodner at
Bell Laboratories, Victor Steinberg in Israel, Ingo Rehberg in Germany, Sergio
Ciliberto in Italy (now France), Carlos Perez-Garcia in Spain, Vincent Cro-
quette, Alain Pochet, Eduardo Wesfreid, and others in France, and Shoichi
Kai in Japan all set up laboratories for the study of patterns in RBC, in electro-
convection in nematic liquid crystals, in TVF, or in other pattern-forming sys-
tems.

At the beginning of 1980, I moved from Bell Laboratories to the Univer-
sity of California at Santa Barbara. After devoting a year or so to setting up
my low-temperature laboratory, I returned to the study of non-equilibrium sys-
tems. In collaboration with my colleague Dave Cannell, we started a program of
investigations of non-equilibrium fluid-mechanical systems at or near room tem-
perature. Dave had tremendous experience from his work on critical phenomena
in the control of temperature, design of complex apparatus, and precision mea-
surements especially by optical means. For instance, it turned out to be possible
to control the temperature of a water bath within about $10^{-4}^\circ\text{C}$, [79] which is
0.3 ppm of the absolute temperature and in that sense within a factor of three
or so of what at that time could be achieved at low temperatures. Initially, we
focused on RBC, using water as the fluid. During this time Victor Steinberg
was a postdoctoral researcher in our laboratory and played an important role
in the development of our instrumentation. Soon we added Taylor-vortex flow
(TVF) because it offered exceptional opportunities for the quantitative study of
one-dimensional patterns. One of its great virtues is that it is truly periodic in
the $y$-direction (i.e. in the azimuthal direction). This is an advantage over RBC
when it comes to comparing with simple mathematical models which often as-
ume periodicity in that direction. TVF had of course been used very extensively by many others before, including Koschmieder, Cole, Snyder, Gollub, Swinney, Donnelly, and their numerous coworkers. [80] But much of that work had focused on somewhat higher Reynolds numbers where secondary bifurcations and chaotic regimes exist, whereas we concentrated more on a quantitative study near the primary bifurcation of the Taylor-vortex state itself. Dave had designed and our machine shop had built an excellent laser-doppler velocimetry system (with the limited funding which we were able to attract we could not afford a commercial one), which served us well in this work.

One of the hallmarks of this decade was the study of advanced bifurcation phenomena, and their application to spatially extended systems. One could reasonably ask whether all the phenomena allowed by the normal forms of bifurcation theory really occurred in physical systems. The answer that became apparent was that certainly very many of them did. Of interest to mathematicians at this time was the area of multi-parameter bifurcations. In 1985, an entire conference in Arcata, CA was devoted to this topic. [82] When there are two control parameters which can be adjusted, it is sometimes possible to choose the value of one so that the bifurcation provoked by a change in the other is of a special nature. For instance, it may be possible to adjust a parameter so that the coefficient $g_3$ of the cubic term in the Landau equation relevant to a particular

![Image of diagrams showing order parameter $\Psi$ as a function of $\epsilon$ for three aspect ratios $L$, and the coefficient $g_3$ as a function of $L$. Adapted from Ref. 81.](image)
system is zero or negative. In that case, the next-higher order (quintic) term has to be retained in the amplitude expansion in order to obtain a stable solution. For the marginal case \( g_3 = 0 \) the amplitude of the flow is expected to grow as the fourth root of \( \epsilon \) instead of as the second. Based on the work of Benjamin and Mullin [83], we saw that this phenomenon should occur in the flow between concentric cylinders when the aspect ratio \( L \) (the system length measured in units of the gap between the cylinders) has a particular small value. For \( L \) less than about two, there are only two vortices. Since these are provoked by the cylinder ends, they are not really Taylor, but rather Ekman vortices, and the system might be referred to as Ekman-vortex flow. As the inner-cylinder speed \( \omega \) is increased beyond an \( L \)-dependent value \( \omega_1 \), the system undergoes a bifurcation from a state of two symmetric vortices of equal size to a state of broken symmetry where one vortex is larger than the other. One can define an order parameter \( \Psi \) which measures the asymmetry between the two vortices. As \( L \) is varied from larger to smaller values through a special value \( L_t \), the bifurcation changes from supercritical, where \( \Psi \) grows continuously from zero as \( \epsilon = \omega/\omega_1 - 1 \) is increased through zero, to subcritical where \( \Psi \) jumps from zero to a finite value near \( \epsilon = 0 \).

The marginal case \( L = L_t \) is often called tricritical, in analogy to the equivalent phenomenon in equilibrium critical phenomena. With Anneli Aitta, [81] a visitor from Finland in our group, we investigated this system quantitatively by laser-doppler velocimetry measurements of the axial component \( v_z \) of the fluid velocity as a function of axial position. Figures 4a to 4c show the experimental results for steady-state values of the order parameter \( \Psi \equiv \int_0^L v_z dz / \int_0^L |v_z| dz \), together with fits to the Landau equation \( d\Psi/dt = h + \epsilon \Psi - g_3 \Psi^3 - g_5 \Psi^5 = 0 \). Here the "field" \( h \) was included because the bifurcation in the physical system was not quite ideal, but rather "imperfect". We actually viewed the imperfection positively, since it gave us the additional opportunity to study the effect of an imperfection on bifurcations of different types. An interesting consequence of having \( h \neq 0 \) is the unfolding of the bifurcation. In the experimental results, the two branches became disconnected, as they should according to the model. As can be seen, the fits to the Landau equation are excellent. They provide the coefficients of the equation as a function of \( L \). The coefficient \( g_3 \) of the cubic term is shown in Fig. 4d. The tricritical value of \( L \), where \( g_3 = 0 \), turned out to be 1.255. At about the time of our experiments, the bifurcation diagram for this system was calculated by numerical integration of the Navier-Stokes equation. [84,85] This work yielded [84] \( L_t = 1.259 \), in quantitative agreement with the experiment. A bit later, Anneli extended the analysis of her data to the transients of \( \Psi \), i.e. to the dynamics of these bifurcations. [86] This provided beautiful illustrations of the difference in the temporal evolution of \( \Psi \) at fixed \( \epsilon \) which is associated with the different shapes of the potentials for the subcritical and the supercritical bifurcation.

Another multiparameter-bifurcation problem under investigation in our laboratory at that time [87,88] was the codimension-two bifurcation which occurs in convection of binary mixtures. [89,90] This work was done by Ingo Rehberg, who was a postdoctoral researcher in our group. Ingo’s system consisted of a
rectangular cell of width 4 and length 8 times the thickness. In order to approximate a porous medium, the cell was packed with a bodycentered cubic array of over 1000 nylon spheres and was filled with a mixture of $^3\text{He}$ and $^4\text{He}$. By changing the mean temperature near 2.2 K, the separation ratio $\Psi$ could be changed with great resolution in the vicinity of $\Psi_{ct} \simeq 0$ where according to the theory a Hopf-bifurcation line should meet a line of subcritical bifurcations to a steady pattern. This phenomenon was indeed observed. In the experiment it turned out that the Hopf frequency approached zero as $\Psi$ was increased, as had been predicted. Perhaps even more exciting, the characteristic frequency at constant $\Psi$ decreased towards zero as $\epsilon$ was increased, as was expected upon approaching a line of heteroclinic points which had been predicted to extend from the codimension-two point towards more negative $\Psi$. To this day it has not been possible to repeat this experiment in a room-temperature system because $\psi$ cannot be varied so easily and with such resolution. Thus, here is a case where the extra effort and cost of the low-temperature experiment was justified in spite of the disadvantage of the lack of flow visualization.

We extended the cryogenic-mixture work also to a bulk fluid (i.e. without the nylon spheres) in a larger cell of width 6.5 and length 26. [91] This work led to the unexpected conclusion that the bifurcation to steady convection apparently has a tricritical point at positive $\Psi$, a result which is difficult to understand and as yet not explained. This work was continued by Tim Sullivan, [92,93] a new postdoctoral associate in our group, after Ingo joined the institute of Professor Busse in Bayreuth, Germany. Tim’s results show that the Hopf frequency at the codimension-two point of the bulk mixture is at least an order of magnitude greater than had been calculated. [94−96] It is clear that the codimension-two point in the bulk mixtures is not well understood at this time, and well worthy of further investigations. One may hope that in the future room-temperature techniques can be refined sufficiently to shed some light on the problem.

Although the work on multi-parameter bifurcations which I have described was very exciting, it is perhaps fair to say that another area of activity of ours, involving stability ranges and wavenumber adjustments of spatially extended systems, had a greater impact on the field of patterns in non-equilibrium systems. We became involved in this on the one hand because of our interests in studying the TVF state. But on the other, a great impetus was provided by a program at the Institute for Theoretical Physics (ITP) at Santa Barbara in 1982 which was run by Pierre Hohenberg and Jim Langer. This program brought many of the theorists active in pattern formation to town, and in addition some who were just thinking of getting into this field. It was well known at this time that there was a continuous band of stable states above a typical primary bifurcation from a spatially uniform state, limited in the case of TVF by the Eckhaus instability. [97] A particularly interesting issue for us which evolved from the ITP Program was the question of whether a spatial ramp in the control parameter from below to above critical would allow the selection of any one of the states within the stable band, or whether a particular state would be preferred over the others. Theoretically, this issue was discussed, and illustrated in the context
of nonlinear reaction-diffusion equations, in a paper by Lorenz Kramer, Eshel Ben-Jacob, Helmut Brand, and Mike Cross, [98] all of whom were at the ITP. As the result of many stimulating lunchtime discussions at the Arbor on our Campus, we started an experimental program to address this issue. During the next several years, an intense and very fruitful interaction between the theorists and experimentalists developed which, I think, brought much reward to all involved. The experiments soon revealed a unique wavenumber when the outer cylinder of the TVF apparatus had a gentle ramp in its diameter. [99] Here I might say that it is a great tribute to our machine shop and to the skills of Rudy Stuber (the head of our shop) that the required precise geometries [100] could be produced. After the initial excitement, the issue of stability ranges and selection processes became a serious program for us. Marco Dominguez-Lerma, a graduate student in our group, designed and built a superb TVF apparatus with a near-perfect geometry. He made highly quantitative measurements of the Eckhaus-instability line in the $\epsilon$-wavenumber plane. Obviously, this was necessary before any selected states could be appreciated. Figure 5 shows his results. [100,101] Although the Eckhaus boundary had been investigated before in a number of systems, [102 – 104] really only the essentially simultaneous work by Lowe and Gollub [104] on electroconvection in a nematic liquid crystal can be regarded as quantitative. Marco’s measurements were in excellent agreement with calculations based on the NS equations and a Galerkin method by Riecke and Paap [105] which are shown by the solid line in Fig. 5. Figure 6 shows again the Eckhaus boundary, and in addition as plusses the data obtained by Marco [99,100] for the state selected by a ramp in the outer-cylinder radius.

![Graph](image_url)

Although the investigation of wavenumber selection by ramps was at least in part motivated initially by a hope of finding something equivalent to an extremum principle for these nonequilibrium systems, it soon became clear from the theoretical work [98,106,107] that the unique wavenumber had a different origin. Somewhere along the ramp, one has $\epsilon = 0$, and at that point only the critical wavenumber can occur. From there on into the interior of the system.
For Two Decades of Pattern Formation, a Personal Perspective

Fig. 5. The Eckhaus instability boundary for TVF. The solid points are from Ref. 100. The solid line is the calculation by Riecke and Paap. [105].

Fig. 6. The Eckhaus instability boundary for TVF as in Fig. 5, and the predictions for the selected wavenumbers for the three different ramps in the cylinder radii which are shown schematically at the top of the figure. The plusses are the experimental results from Ref. 100.

(Where $\epsilon > 0$) the axial variation of the phase of the pattern, and thus the change in its gradient which is the wavenumber change, is fixed by the equations of motion. This spatial phase variation could be calculated conveniently using a phase-equation approach. [107]

Fig. 7. Experimental results in the stable range $\epsilon < 0.2$ for the wavenumbers selected by ramps in the inner and outer cylinders with $\alpha_o/\alpha_i = 2$ (case III). Adapted from Ref. 108.

Fig. 8. The frequency of the travelling wave in the ramped section and of the vortex-pair loss in the interior. The lines are the predictions for two slightly different geometries. Adapted from Ref. 108.

The above realization also soon led to the prediction that the selected state depends on precisely how $\epsilon$ is varied from below to above critical. Thus, for instance, in the TVF case one could have a ramp in the outer or inner cylinder, with ramp angles $\alpha_o$ and $\alpha_i$ respectively. Riecke and Paap [107] predicted that the selected state depends upon $r = \alpha_o/\alpha_i$, and that by changing this ratio, any wavenumber inside the Eckhaus-stable band could be selected. The three cases $r = 0$, $r = \infty$, and $r = 2$ are illustrated schematically in the top portion of Fig. 6, and the predicted wavenumbers are given in Fig. 6 as cases I, II, and III respectively. Case III is particularly intriguing. Here the prediction is that the selected wavenumber should be unstable for $\epsilon \gtrsim 0.2$. This case, as well as case I,
was investigated experimentally by Li Ning, a student in our group. [108] Some of Ning’s results in the stable range for case III are shown in Fig. 7. They agree well with the prediction. For case III and $\epsilon > 0.2$, the selection of the unstable state was predicted to lead to a repeated occurrence of the loss of a roll pair via the Eckhaus mechanism in the system interior. It was expected that these transitions would lead to a travelling wave of vortices coming up the ramp so as to provide the supply of vortices needed to sustain the vortex-pair losses. This is precisely what Ning found in his experiments. Figure 8 shows the frequency of the vortex-pair loss, together with a calculation of this frequency based on the phase equation by Riecke and Paap (the two curves are for two slightly different geometries and represent the uncertainty in the dimensions of the apparatus). We see that even the creation of a *dynamic* state via the selection of an *unstable* state could be calculated quantitatively using phase equations. Thus, even though the search for an extremum principle was in vain, the results of the joint theoretical and experimental effort may well be regarded as a significant advance in our fundamental understanding of patterns.

An interesting related experiment during this time was one of the first in the laboratory of Ingo Rehberg in Bayreuth, where a Rayleigh-Bénard system with different ramps on the two ends was investigated. [109] The two ramps selected different wavenumbers, and thus created a wavenumber gradient in the system interior. From the phase-diffusion equation this is expected to lead to a travelling wave of convection rolls, which was indeed observed in the experiment. The travelling wave is shown in the right portion of Fig. 9, which is a contour plot of the pattern amplitude as revealed by a shadowgraph method as a function of position and time. For comparison, the left portion of Fig. 9 gives a similar contour plot for Ning’s experiment on Case III and for $\epsilon = 0.30$. Here the ramp is at negative values of the axial position $z$, and contains the travelling wave. Vortex-pair losses occur in the uniform supercritical section at the position indicated by the arrow.
One of the fascinating topics which became popular in this decade was the study of convection in binary mixtures. I already mentioned the cryogenic work near the codimension-two point. Of much broader scope was work at room temperature with flow visualization which evolved after the experiment of Walden et al. [110] made it generally known that this system sustains travelling waves of convection rolls. The papers which appeared during the subsequent few years are simply too numerous to be mentioned here. Major contributions came from Kolodner and Surko at Bell Laboratories. Victor Steinberg, after moving from Santa Barbara to Rehovot, Israel early in 1984, built convection apparatus very similar to ours and started binary-mixture work. The field was particularly exciting because of the very close interaction between experimentalists and theorists. The theoretical work of Cross, Knobloch, Lücke, Brand, Deissler, and others on some occasions was stimulated by experiment, and on others motivated new measurements. At Santa Barbara, we initially studied ethanol-water mixtures in narrow rectangular containers with Richard Heinrichs, a post-doctoral associate in our group. Richard made the remarkable discovery that the travelling waves which evolve from the conduction state can focus into a stable localized pulse. [111, 112] The pulse has a stationary or very slowly moving envelope, and travelling convection rolls form at one end and leave it by decaying to zero amplitude at the other. The same discovery was made at about the same time and quite independently in Israel by the Steinberg group. [113] Naturally there were some skeptics who suggested that the very existence of these pulses might be attributable to the short sidewall towards which the waves were travelling. Thus we as well as others [114] immediately wanted to do experiments in annular containers which lack the short sidewall of the rectangular cell. Just about then we went through some extremely frustrating times because the NSF Engineering Directorate decided to stop funding the kind of work which we physicists were doing. Our group, Gollub, Swinney, and Donnelly were all eliminated from their program in spite of excellent reviews of our proposals. To me at least this appeared as one of the low points in the history of funding of fundamental research in the United States. After considerable delay, we were able to get support from another agency with a more enlightened policy. But of course it took time to get going again because new personnel had to acquire the skills to do these sophisticated experiments. In the end we were able to show that the pulses existed even in a periodic system. [115] The left portion of Fig. 10 gives a shadowgraph image of a three-pulse state in this geometry which was obtained by Joe Niemela in our group. The experimental discovery of pulses let to considerable theoretical activity. Soon after they were found in the experiments, pulses were observed numerically by Thual and Fauve [116, 117] as a solution of a Ginzburg-Landau equation with a destabilizing cubic and a stabilizing quintic term. Such an equation might be regarded as a reasonable approximation (albeit...
not a systematically derived envelope equation) for binary-mixture convection. Much further interesting numerical work based on model equations was carried out by Brand and Deissler. [118, 119] Lücke and collaborators studied pulses by the numerical integration of the Navier-Stokes equations for binary-mixture convection. [120] It has been suggested that pulses are related to the solitons of the nonlinear Schrödinger equation, modified by the dissipative contributions to the Ginzburg-Landau equation or the physical system. [121] A detailed theoretical discussion of fronts and pulses was given by van Saarloos and Hohenberg. [122] These localized structures certainly are one of the more interesting pattern-formation phenomena which I have encountered.

Although chronologically it really belongs into the next chapter, I mention here that we extended the study of pulses in binary mixtures to a two-dimensional system. This is work by Kristina Lerman, a student in our group. [56, 123] One of her two-dimensional pulses is shown in the right portion of Fig. 10. They formed spontaneously from the conduction state when the threshold for convection was exceeded by a fraction of a percent. This seemed particularly interesting because it is known that the two-dimensional nonlinear Schrödinger equation does not have solutions which are localized in two dimensions. Here it is interesting to note, however, that long-lived or stable pulses were found numerically by Deissler and Brand [124] in two-dimensional model equations, although these equations are quite different from ones which might represent the experiment on RBC in binary mixtures. Kristina’s pulses turned out not to be stable in the long term. After existing for many hours (the vertical diffusion time is only a minute or so), they split into two parts via a transverse instability at their tail end where the travelling rolls formed. Often one half survived and reformed into a pulse while the other decayed; but eventually a more complicated structure of disordered travelling convection rolls developed. So far as I know, the formation of two-dimensional pulses is not understood theoretically at this time.
I have described a lot of progress in our understanding of one-dimensional patterns. However, much was learned also with RBC in samples which were extended in two dimensions. Particularly noteworthy are the important contributions by Vincent Croquette and coworkers, [53, 125] who studied RBC in compressed Argon with a Prandtl number $\sigma \simeq 0.7$. This is a range not accessible in classical liquids, but similar to $\sigma$ in the early liquid helium experiments. However, in Vincent’s experiments it was possible to visualize the flow. He studied concentric rolls stabilized by sidewall forcing, as well as straight-roll patterns, in cylindrical cells with radius-to-height ratios $\Gamma = 7.66$ and 20. For the "straight" rolls in a small-$\Gamma$ cell, there were two focus singularities bracketing the rolls. The rolls tended to bend toward the singularities as $\epsilon$ was increased, resulting in compression of rolls along the line connecting the singularities. When the compression became too large, the rolls in the center of the cell, where the compression was most severe, underwent a skewed-varicose-like instability. For $\Gamma = 7.66$, this first occurred at $\epsilon = 0.13$. In the large-$\Gamma$ case, the onset of time dependence was at the lower $\epsilon = 0.085$. One compressed roll pair got pinched off and nucleated two defects. These two defects climbed along the roll axis to within 1 or 2 wavelength of the sidewall and then glided along the wall to disappear into the focus singularities. When the defects disappeared, the singularities nucleated a new roll pair, resulting in a pattern containing the same number of rolls as before. This process then repeated itself. It seems likely that similar phenomena were responsible for many of the various time dependences observed in the early liquid helium experiments. [38, 39, 52] The onset of this instability has been demonstrated to be strongly dependent on large-scale flows which are associated with roll curvature. [126] When the flow was suppressed, the tendency for the rolls to end more perpendicular to the sidewall at higher $\epsilon$ was also suppressed, and the straight-roll state was observed to be stable to $\epsilon$ as large as about 0.6.

Finally I want to mention just briefly the very interesting work on pattern formation in electro-convection (EC) in nematic liquid crystals (NLC) which experienced an upsurge during this decade. The system consists of a thin layer of a NLC confined between transparent plates which are covered with transparent electrodes. The director is aligned uniformly in a specified direction, usually parallel to the confining plates. This is accomplished by appropriate treatment of the surfaces. For parallel alignment, a voltage applied to the electrodes will induce hydrodynamic flow if the dielectric constant has a negative anisotropy and if there are ionic impurities present in the sample. In contrast to the RBC case, which is invariant under rotation in the horizontal plane, this system has a preferred direction and for that reason exhibits quite different pattern-formation phenomena. Important work in this area was done by Kai, Rehberg, Ribotta, Steinberg, and their collaborators. [127, 128] They studied the orientation of the convection rolls relative to the director, stationary and travelling patterns,
defect-chaos states, and many other interesting effects. This system has the potential for quantitative contact between experiments and weakly nonlinear theories, i.e., Ginzburg-Landau equations. [128, 129] The equations of motion are much more complicated than the NS equations of RBC, and for that reason this system provides a more severe test of modern theories of pattern formation. However, there are also more factors such as the conductivity of the sample which can have a decisive influence on the phenomena which occur. Often the experimental foundation for a quantitative comparison with the theory has not really been provided. Usually the experimentalists did not adequately characterize their samples and they did not obtain convincing evidence about the nature of the primary bifurcation which occurred in their samples. Thus, although much has been learned already, there is tremendous potential here for the quantitative study of more advanced pattern-formation phenomena.

4 The 1990’s

The present decade naturally has brought further technological advances to the experimental study of pattern formation. Most of these were again in the data-processing area. Even with a modest budget, we can now have faster computers (486’s) in the laboratory than ever before, and disk storage has become even more affordable. It is now common to take time series of two-dimensional images and to play them back as movies, whereas a decade ago only time series of one-dimensional images were readily accessible to us. This turns out to be a very important capability. Particularly in RBC, many processes are very slow, and within a human attention span a pattern may seem steady. Playing back a movie at a rate much greater than real time often reveals interesting dynamics which might have been missed otherwise. Also the digital data processing has advanced significantly. For instance, it is now well within the reach of the average experimentalist to compute three-dimensional Fourier transforms of two-dimensional image time-sequences on his own work station. So he can extract $S(k, \omega)$ from the data. A decade ago we did not have this capability.

One of the new directions in our laboratory was the study of convection in gases under pressure. As I described above, this had been pioneered by Vincent Croquette a few years earlier. [53, 125] John deBruyn started to develop and his successors Eberhard Bodenschatz, Stephen Morris, Yu-Chou Hu, and Mingming Wu perfected a rather complicated apparatus for the study of gas convection at pressures up to 100 bar or so. [130] It differed from Croquette’s apparatus in that the water bath outside the sample cell was also pressurized, thus providing hydrostatic conditions for the entire cell. With this design there was no pressure differential across the cell top and bottom, and we were able to achieve large-diameter cells (up to 9 cm) with no distortion of the confining top and bottom surfaces. Our interest at first was in finding a binary mixture of gases with a negative separation ratio. This would have had the advantage of a
Lewis number of order one rather than $10^{-2}$, and for instance would have made codimension-two phenomena much more accessible to experiment. The transport properties of gas mixtures are not very well known, and so it was difficult to find an appropriate mixture. While we were searching, we became discouraged in our effort by a theoretical development. It turns out that in gas mixtures the Dufour effect [131] becomes important. Calculations by Hort et al. [132] showed that it would be unlikely that travelling waves would exist near a codimension-two point. Nonetheless, there would be interesting phenomena to be explored; but for the present at least we let ourselves be distracted by a variety of other fascinating problems involving RBC in pure gases.

One of the aspects of convection in gases is that a change in the pressure will produce a dramatic change in $\Delta T_c$. Alternatively, the same $\Delta T_c$ can be achieved with a range of sample thicknesses $d$ by varying the pressure. This flexibility meant that thin samples with radius to height ratios of order 100 and with extremely short time scales (vertical diffusion times of order one second) could be obtained. So it was possible to study two-dimensional stationary processes in samples of unprecedented size, hopefully approaching the theoretical ideal of a laterally unbounded system. Varying $\Delta T_c$ provided the opportunity to vary the extent of departures from the Boussinesq approximation.

Fig. 11. Left and Middle: One- and two-armed giant spirals at $\epsilon = 0.12$ (Ref. 55) and 0.15 (Ref. 3) respectively. Right: Spiral-defect chaos for $\epsilon \approx 0.7$ (Ref. 133).
Eberhard Bodenschatz developed and used our apparatus to study the formation and stability range of hexagons due to non-Boussinesq effects. [3] It turned out that the hexagonal lattice of convection cells which is stable above onset is perfect so far as we can tell. Any defects which may form initially when $\Delta T_c$ is exceeded migrate to the sample perimeter and disappear. A portion of one of Eberhard’s images was shown already in Fig. 1 at the beginning of this paper. The entire image contains about 5000 convection cells, forming an essentially perfect lattice. So far as we know, this is the largest defect-free dissipative structure that has been produced in the laboratory. Although still small on the scale of crystal lattices in solid-state physics, it suggests that "Broken Symmetry" [134] may exist after all in nonequilibrium dissipative systems. The hexagons were found to be stable over a range of $\Delta T$ which was consistent with the calculations by Busse which I mentioned near the very beginning of this paper. [17] When $\Delta T$ was increased beyond the stability limit of the hexagons, Eberhard found that, after transients had decayed, a single giant spiral filled his sample cell. The spiral could have a number of arms, and after many turns away from the center each arm terminated in a dislocation. The left and middle portions of Fig. 11 illustrate such structures with one and two arms. The giant spirals rotated slowly, with a characteristic frequency of order $10^{-3}$ when time is measured in terms of the vertical diffusion time. Interestingly, the outer terminations of the arms and the tips rotated synchronously, and thus the entire structure was stable. Little is known about these interesting patterns, and serious theoretical and much more detailed experimental work on them surely would be warranted. For instance, one would like to know whether the radial location of the dislocation has a unique value for given conditions, and whether the rotation is caused by a wavenumber gradient in the radial direction or by large-scale flows induced primarily near the tip. It seems to me that these problems fall within that category I mentioned earlier of being complicated enough to be interesting and simple enough to be soluble. There are many similarly interesting issues associated with these large-scale structures which should be pursued. One of them is the temporal behavior of a defect deliberately introduced into the otherwise perfect hexagonal lattice. This defect is known to be associated with two of the three sets of rolls which make up the hexagons. Since these are not orthogonal to each other, the defect can neither climb nor glide. We know that in time it moves out of the system, and its motion must be a combination of climbing and gliding. Will it undergo some sort of irregular chaotic diffusive motion?

Stephen Morris was Eberhard’s successor as a postdoctoral associate. During a period of overlap they worked together and made a remarkable discovery. [133] When a large Boussinesq RBC system of radius ratio 78 and with a Prandtl number near one is taken about 26 % above onset, the pattern turns into a state consisting of many small spirals and other defects which interact with each other and form a state which we call spiral-defect chaos (SDC). A typical snapshot of SDC is shown in the right portion of Fig. 11. Stephen made extensive measurements of its statistical properties. Its mean wavenumber as a function of $\epsilon = \Delta T/\Delta T_c - 1$ falls right into the middle of the wavenumber band over
which, according to the theory, straight rolls are stable. In the end we concluded that there are two attractor basins for the solutions to the equations of motion of the system. One is the straight-roll attractor of Busse and Clever. The other, found in our experiments, is a previously unknown chaotic attractor corresponding to SDC. Much of the experiment has recently been reproduced numerically, both in model equations and by integration of the Navier-Stokes equations in the Boussinesq approximation. I think the discovery of SDC fundamentally changes our view about RBC in systems with .

Previously, we had known that there was slow time dependence in this system; but this was thought to be provoked by locally (in the cell interior) exceeding the skewed-varicose instability because of roll “pinching” due to the curvature of the rolls which in turn was caused by the lateral walls of the cell, and by the emission of rolls from sidewall foci. Now it appears that SDC has nothing to do with the walls and instead corresponds to a stable attractor with a large attractor basin even for the a system with periodic boundary conditions, and presumably for the infinite system.

In a collaboration with Bob Ecke at Los Alamos, we meanwhile had duplicated our gas-convection apparatus and started a second set of experiments. Our student Yu-Chou Hu took up residence in Los Alamos, and carried out this work. Yu-Chou’s primary goal was to study convection in the presence of rotation, which leads to the Küppers-Lortz instability and which was interesting to us because it gives spatio-temporal chaos at onset via a forward bifurcation. This work is not yet completed, although preliminary results have already provoked interesting theoretical work by Tu and Cross. In order to understand the rotating state, we felt it necessary to study also the system without rotation, although we expected it to be quite well understood. Yu-Chou developed pattern-analysis methods which could be used to determine the onset of SDC objectively from the pattern statistics. He found that in the smaller system with a radius ratio occurred for whereas in the system with it had been observed all the way down to . This is consistent with the fact that Croquette, in a system with the smaller , did not report any SDC even though he worked up to quite large -values. The question naturally arises of how low the onset of SDC would be in the laterally infinite system. Clearly, the determination of the aspect-ratio dependence of the SDC onset is an interesting problem for the future. One would also like to understand why the onset is so sensitive to when is already so large. Presumably the answer to this latter question will be that sidewalls suppress mean flows, and that mean flows are essential to the sustenance of SDC. Interestingly, Yu-Chou found that the onset of SDC occurred at somewhat smaller when the system was rotated around a vertical axis. In addition, rotation broke the chiral symmetry of the spirals in the sense that the numbers of right-handed and left-handed spirals in the SDC state were no longer equal.

I believe that one of the experimental accomplishments of this decade is that we are finally learning about the influence of thermal noise on macroscopic hy-
drodynamic systems. From a theoretical viewpoint, this has been an active issue in the 1970's, [57–59] but as I mentioned above, the effects were generally considered too small to be observable in real experiments because the energy $k_B T$ is many orders of magnitude smaller than typical energies involved in macroscopic fluid motion. Nonetheless, by judicious choices of experimental systems it has now been possible to observe and measure directly the consequences of fluctuating fluid-velocity fields near bifurcations which are the response of the system to thermal noise. The first measurements were made in electro-convection in a nematic liquid crystal. [144] In that system the conditions are particularly favorable because the thickness of the fluid layer, and thus the typical fluctuating volume, is small. In addition, the relevant macroscopic dissipative energy is determined by the Frank elastic constants, [145] which are also relatively small. [146] In the liquid crystal, there is a preferred direction corresponding to the average alignment of the molecules, and this is reflected in the fluctuating convection rolls. Thus, the wavevectors of these rolls are centered about two distinct positions in the two-dimensional Fourier space. More recently, quantitative measurements of the fluctuation amplitudes in RBC have been made, using compressed CO$_2$ as the fluid. [63] In this case, the system is isotropic in the plane, and therefore wavevectors of all possible orientations but with a modulus close to the critical wavenumber are equally represented in the fluctuations. It turns out that the fluctuation amplitudes vary as $1/|\epsilon|^{0.25}$, precisely as predicted on the basis of a stochastic GL or Swift-Hohenberg equation with white noise. The experimentally measured amplitude of this power law agrees within 20% or so with predictions [61, 62] based on the formulation of this problem by Landau and Lifshitz [131].

An important characteristic of the present decade is that physicists are beginning to tackle more complicated, and thus in some sense more practically relevant, systems. As I described above, the previous two decades had been used to establish a firm foundation for this. One of the interesting directions being pursued is open flows. The particular case of TVF with an imposed axial flow has been a most fruitful example for quantitative laboratory study. [147–153] Here too noise plays a central role. In the absence of noise there is a range of $\epsilon$ and throughflow Reynolds-numbers over which, although disturbances have a positive growthrate, the throughflow velocity is large enough to sweep away an isolated disturbance faster than it can grow. In this “convectively” unstable regime, there would be no spatial structures if they were not continuously provoked by external noise. It turns out that even microscopic noise can create “noise-sustained structures” [154]. A quantitative explanation of the observed phenomena was possible in terms of a stochastic complex Ginzburg-Landau equation. [148, 149] One may hope that this work will provide guidance in the understanding of more complicated open-flow systems.

Another thrust towards understanding more complex pattern-formation phenomena which is being pursued at present is the study of RBC in nematic liquid crystals. We became interested in this problem when I learned about it during a sabbatical leave in 1990 at the University of Bayreuth, where Professors Kramer and Pesch had started a program of theoretical analysis of this system. The
equations of motion are significantly more complicated than the Navier-Stokes equations for isotropic fluids. The usual viscosity and conductivity are replaced by five independent viscosities and two conductivities, and the equations for momentum and energy balance must be coupled to an equation for the director field which contains three elastic constants. In spite of these complexities, it has been possible to carry out quantitative stability analyses of these equations, and under some conditions predictions in the weakly nonlinear regime have been made. [128, 155, 156] The system is very rich since the director (average alignment direction of the molecules) can be horizontal (known as parallel alignment) or vertical (known as homeotropic alignment). In the homeotropic case convection can occur when heating from below or above. In addition, a magnetic field in the same direction as the director can be used as a second control parameter and has a profound effect on the expected phenomena. Several of the predictions have been confirmed by recent experiments. [157] For instance, the measured critical Rayleigh numbers $R_c$ as a function of the magnetic field strength $H$ agree quantitatively with the predictions for both the parallel and the homeotropic case. In the parallel case, there is excellent agreement between experiment and calculations for the orientation of the convection rolls relative to the field and the director. Many opportunities exist in this system for additional interesting experiments, and some of these are under way.

5 Outlook

At this point of a review it is customary to make predictions of major breakthroughs which will serve to solve the primary outstanding problems. I have no reason to believe that these are likely to occur. The account which I have given of the last two decades or so has been one of gradual progress. In the 1970’s we considered the simplest possible problems, namely how time and length scales evolve as the threshold of a forward bifurcation is crossed. [28] Some issues of temporal complexity were raised in spatially extended systems, [37, 38] but really no progress towards their elucidation was made. In the 1980’s we built upon what had been learned, and gained more insight into complicated bifurcation phenomena [81, 87] and into the simplest stability limits of spatially extended systems [100]. During the end of the last and the beginning of the current decade we have learned about non-trivial spatially-extended structures such as pulses and fronts, [111, 113, 115] we have made great progress in understanding textured patterns, [4, 53, 54, 125] and we have made an attempt to penetrate the secrets of spatio-temporal complexity [133]. I think that this gradual process will continue. As in the past, some will rush ahead and tackle problems well beyond current comprehension; but others will advance more cautiously and study the next-more complicated problem on the horizon in detail. I hope that this work will include further experiments on open systems which can be well controlled in the laboratory, [149] because these are important not only from the scientific viewpoint, but also in relation to practical issues involving industrial
processes and engineering applications. I also hope that our understanding of patterns in more complex fluids will increase. A step in that direction is the recent work on RB- and electro-convection in nematics. I hope that this will be continued, and extended to such systems as ferrofluids and visco-elastic fluids. One of the Holy Grails is of course, as it has been in bygone years, the elucidation of spatio-temporal chaos. Are there universality classes which may serve to categorize these complex phenomena? Can these effects be captured in the solutions of simple model equations such as complex Ginzburg-Landau and Swift-Hohenberg equations? Will the much more advanced understanding of dynamical systems be helpful here? To what extent do stochastic effects which come from outside the usual hydrodynamic equations play a role? These are some of the issues which I expect to be addressed during the next decade, albeit one step at a time lest we should get lost at dead ends. In more concrete terms and from the experimentalist’s viewpoint, I am hopeful that the phenomena observed in hydrodynamic instabilities in nematic liquid crystals [127, 128, 158] and perhaps some other anisotropic systems, which a priori appear much more complicated than RBC in isotropic fluids, will in a sense provide a simplification by virtue of the preferred orientation. Breaking the rotational symmetry of RBC in isotropic fluids, although it leads to mathematically more complicated equations, may well lead to simpler physical phenomena. As an example I mention here the STC discovered recently by Mike Dennin in our group and illustrated in Fig. 12. [158] In electro-convection in a particular liquid crystal (I52 over the right temperature range and appropriately doped at the right level) one

![Fig. 12. Spatio-temporal chaos in electro-convection (Ref. 158). The image on the left is Fourier-decomposed into its two components in the middle and on the right.](image)

obtains a forward bifurcation to a state of STC. The state consists of travelling convection rolls which have an oblique orientation to the director. In that case, the two modes with angles $\Theta$ and $-\Theta$ are equally likely to occur. Under some conditions, both modes occur simultaneously, on the one hand stabilizing each other and on the other influencing each other’s amplitudes in a complex manner. A state of amplitude-chaos arises where the mode amplitudes will, as time passes,
go through zero at some spatial locations. At these points, defects can form in the roll structure. Since this state arises via a forward bifurcation, one may hope to capture these rich phenomena in a relatively simple model, which may consist of a weakly nonlinear theory involving two coupled complex GL equations, one for each of the two roll orientations. It is my hope that such relatively simple examples of STC will serve to lay a foundation for the gradual classification and understanding of these complex phenomena. If significant progress is made in this direction during the next decade, I believe that we as a community should be very satisfied with our accomplishments. I regard as a very hopeful sign for progress in that direction the encouraging number of young people who have entered the field during the last few years.

REFERENCES

18. Earlier theoretical investigations of non-Boussinesq convection were carried out by several theorists, starting to my knowledge with Palm [19]; but Busse's calculation was the first which systematically included the temperature dependence of all relevant fluid properties.

74. For a description of early work by Silveston [10], see Sect. II.18 of Ref. [75].