Experiments on Spatio-Temporal Chaos

Guenter Ahlers

Department of Physics and Center for Nonlinear Science, University of California, Santa Barbara CA 93106, USA

Abstract

Under non-equilibrium conditions, a spatially-extended system will often undergo a transition from a uniform state to a state with spatial variation (a "pattern"). Patterns are of interest because their formation is associated with *nonlinear* effects which can lead to qualitatively new phenomena that do not occur in linear systems. Among the most fascinating of these phenomena is spatio-temporal chaos (STC). This paper briefly reviews some of the experimental observations of and measurements on STC.

1 Introduction

So far as I can tell, the quantitative study of chaos in *spatially extended* systems (STC) had its origin in experiments on Rayleigh-Bénard convection (RBC) at cryogenic temperatures. [1–4] This early work was followed soon by quantitative measurements [5,6] on *temporal* chaos in systems without significant spatial extent which, for some time, attracted far more attention because they made contact with concurrent theoretical developments [7]; this interaction between theory and experiment revived the field of dynamical systems as a branch of physics. [8] By now this field has reached a certain level of maturity. Here I want to examine some of the experimental results on chaos in systems with significant *spatial* variation. For these the level of theoretical understanding is still much more limited than it is for dynamical systems.

⋆ This paper is based on work done with many collaborators, including K.M.S. Bajaj, R. Behringer, U. Bisang, E. Bodenschatz, D.S. Cannell, M. Dennin, R.E. Ecke, Y.-C. Hu, J. Liu, S.W. Morris, and M. Urisch.

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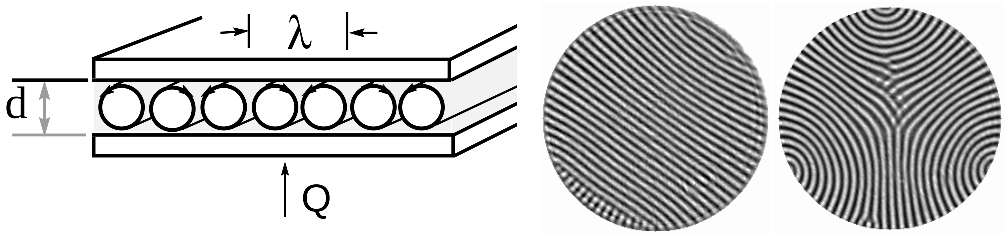

Fig. 1. Left: schematic diagram of Rayleigh-Bénard convection. Middle and right: shadowgraph images taken from above of typical convection patterns. Middle: straight rolls for $\epsilon = 0.07$. Right: A pattern for $\epsilon = 0.2$ where the rolls have become curved and where sidewall foci and defects have been generated (note that no spirals are present at this ϵ). The fluid is compressed Argon gas with a Prandtl number of 0.69 in a cell of circular horizontal cross section with a radius-to-height ratio $\Gamma = 30$. After Ref. [9].

RBC is illustrated schematically in Fig. 1. It occurs in a shallow horizontal layer of a fluid heated from below by a heat current Q when the temperature difference ΔT exceeds a critical value ΔT_c. Near onset and in the absence of boundary forcing, it consists of parallel, straight rolls with a wavelength λ equal to about twice the layer thickness d. Typical patterns [9] are shown in the middle and right of Fig. 1.

Measurements of the time-averaged Nusselt number $\mathcal{N}$ (effective conductivity divided by the conductivity in the absence of flow) during the early cryogenic experiments (for which there was no flow visualization) are shown in Fig. 2a as a function of $\epsilon \equiv \Delta T/\Delta T_c - 1$. A surprise at the time of those measurements was that the convection depended non-periodically on the time t already at the relatively small values $\epsilon \gtrsim 1$. [1–4] This is illustrated in Fig. 2b for a circular cell with an aspect ratio Γ (radius/height) = 5.3 and $\epsilon = 1.23$. The power spectrum of $\mathcal{N}(t)$ was broad, with a maximum at the frequency $f = 0$, and for large f it fell off as f^{-4} as shown in Fig. 2c. The experimentally observed algebraic falloff was surprising because simple models of chaos in deterministic systems with relatively few degrees of freedom, such as the Lorenz model, have a spectrum with an exponential falloff. [10,11] It seems likely [2] that the onset of time dependence was associated with an adjustment of the wavenumber k as a function of ϵ which caused the system to cross an instability boundary, from our present vantage point most likely the skewed-varicose (SV) instability. [12] The apparently algebraic falloff of the spectrum presumably is then attributable to the presence of a large number of interacting modes in the spatially extended system which turns out to lead to effectively algebraic decay over the experimentally accessible range of f; but so far as I know a quantitative explanation of this phenomenon is still lacking. In a qualitative sense this suggestion that many modes come into play as the spatial extent increases is an early indicator that spatio-temporal chaos is high-dimensional, and perhaps *extensive* in the sense that the number of modes (or basis functions) needed to describe it is proportional to (or at least increases with) the system size