

Opportunities for High-Resolution Measurements and Microgravity Research near the Superfluid Transition of Liquid ^4He

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Six years ago an experiment to determine the specific heat of liquid helium near the superfluid transition in a microgravity environment opened up a new era of very-high resolution measurements. Recently this led to the establishment of the Fundamental Physics Discipline by NASA for the support of microgravity research in low-temperature/condensed-matter physics, laser cooling and atomic physics, and gravitation and relativity. This paper describes very-high resolution and microgravity research in one particular sub-field of low-temperature physics, namely the superfluid transition of ^4He , in order to illustrate the diverse research opportunities which exist within the Discipline. The effect of gravity on this system will be illustrated. Projects carried out already and expected to be undertaken in the near future on critical phenomena, surface effects, and non-equilibrium phenomena, will be discussed. These include measurements of the specific heat and the thermal conductivity, both in bulk samples and finite geometries, of the singularity of the boundary resistance between helium and a solid surface, as well as of non-equilibrium effects due to finite heat currents.

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1. Introduction

The superfluid transition of ^4He offers exceptional opportunities for ultra-high resolution measurements. In part these opportunities can be fully exploited only in a microgravity environment. The exceptional opportunities arise for two reasons. First, this system is uniquely suitable for detailed ex-

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perimental study because of the near-perfect nature of the samples available in the laboratory. Since the transition is between two liquid phases, strains which spread out the phase transition in solid systems do not exist. Similarly, impurities which alter the behavior of systems near other critical points are essentially absent because they freeze out at the low temperatures. The one remaining inhomogeneity which limits the details in which many properties of the transition can be studied is the gravitational inhomogeneity present in the Earth-bound laboratory. Its elimination by experiments in microgravity thus provides unique opportunities not realizable in other systems. Then the only limitations to the detail in which a sample can be studied are either the limited skills of the experimentalist, or ultimately the fluctuations inherent in finite systems in contact with a heat reservoir. Present experimental methods are indeed approaching this ultimate limit imposed by statistical mechanics.¹ The second reason for the uniqueness of the research opportunities offered by this system is that the fundamental theoretical framework for a basic understanding of diverse phenomena near this transition is believed to be provided by the renormalization-group theory. There has been, and continues to be, a major effort to produce detailed theoretical results based on this framework. This combination of unique research opportunities both in experiment and in theory deserves the continued attention of the scientific community not only because of the results obtainable for this specific system, but also because it makes the superfluid transition a potential major testing ground of the general theoretical principles and methods which are involved. These principles and methods find applicability in many other subfields of physics, including statistical mechanics, condensed-matter physics, and high-energy physics.

Research designed to exploit the exceptional opportunities offered by the liquid-helium system received a major boost in recent years by an important development within the National Aeronautics and Space Administration (NASA), namely the establishment of a new Discipline for Fundamental Physics (FP) in the Microgravity Research Division (MRD) of the Office of Life and Microgravity Science and Applications (OLMSA). Within MRD there have long been four Disciplines, devoted to research in Fluid Physics, Materials Physics, Biotechnology, and Combustion. The addition of the Fundamental Physics Discipline opens up large areas of new opportunities for fundamental microgravity research in several subfields of physics. At present these areas include Low-Temperature and Condensed-Matter Physics, Laser Cooling and Atomic Physics, and Relativity and Gravitation. But as new ideas evolve for using the microgravity environment for research, it is expected that the Discipline will broaden its scope. Both ground-based research on problems for which the Earth's acceleration is a significant factor

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and research projects in an actual microgravity environment (for instance on the Shuttle or on the International Space Station) are supported. They supplement each other and form a coherent program. Studies of the superfluid transition of liquid helium are merely one aspect of Low-Temperature Physics, and what I describe here is only a small example of the research opportunities offered by the FP Discipline.

Returning now to the particular problem of the superfluid transition, we note that several issues can be addressed in great detail along the λ -line, and many but not all of these require microgravity. Of particular current interest are three separate topics. These are critical phenomena, solid-fluid interfaces and finite-size effects, and non-equilibrium phenomena. Below we will discuss each of these in separate sections. All of them benefit from the special circumstances which prevail near the λ -line.

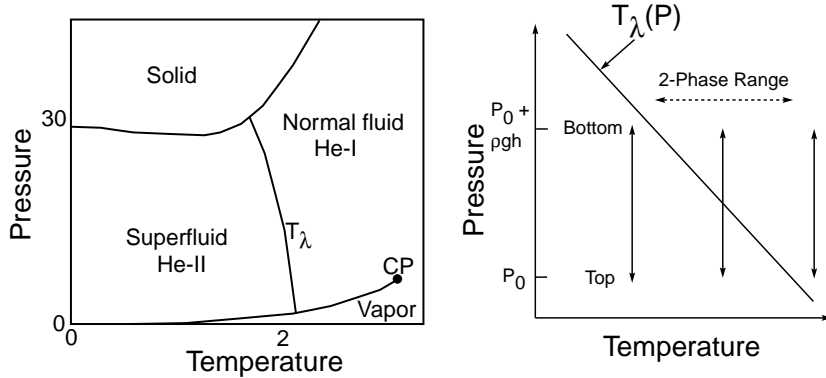


Fig. 1. Schematic phase diagram of ${}^4\text{He}$, and an illustration of the gravity effect.

2. The effect of gravity and modern thermometry

The phase diagram of liquid ${}^4\text{He}$ is illustrated in Fig. 1. There is a line of continuous phase transitions, or critical points, known as the λ -line, along which the liquid undergoes a transition from a superfluid (He-II) to a normal (Navier-Stokes) liquid (He-I). This line extends from 2.176 K at vapor pressure to 1.77 K at the melting pressure of about 30 bar.

The inhomogeneity induced by gravity² is due to the hydrostatic pressure which varies with height in the liquid. This pressure variation has the effect of inducing a vertical spatial variation of the transition temperature. This is illustrated in more detail in the right portion of Fig. 1. If the sample top is at a pressure $P = P_0$ (which might for instance be the saturated vapor pressure), then the bottom will be at $P = P_0 + \rho gh$ where ρ is the fluid

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density, g the gravitational acceleration, and h the sample height. Over this pressure range the transition temperature $T_\lambda(P)$ varies significantly. Associated with the distribution of transition temperatures is a more subtle “averaging” of the properties due to the growth of the correlation length near the *local* transition which no longer fully reflects the properties of a three-dimensional bulk transition. At vapor pressure the variation of $T_\lambda(P)$ is about 1.3×10^{-6} K/cm. Thus, for a typical sample of size 1 cm, it is not possible to approach the transition more closely on average than within about one μ K. This is three or four orders of magnitude larger than the temperature resolution now available in modern laboratories.³ It is clear that the *full* potentials of the superfluid transition can not be exploited in an Earth-bound environment because the gravitational effect sets the limit on the detail in which a variety of questions can be answered by modern experimental techniques.

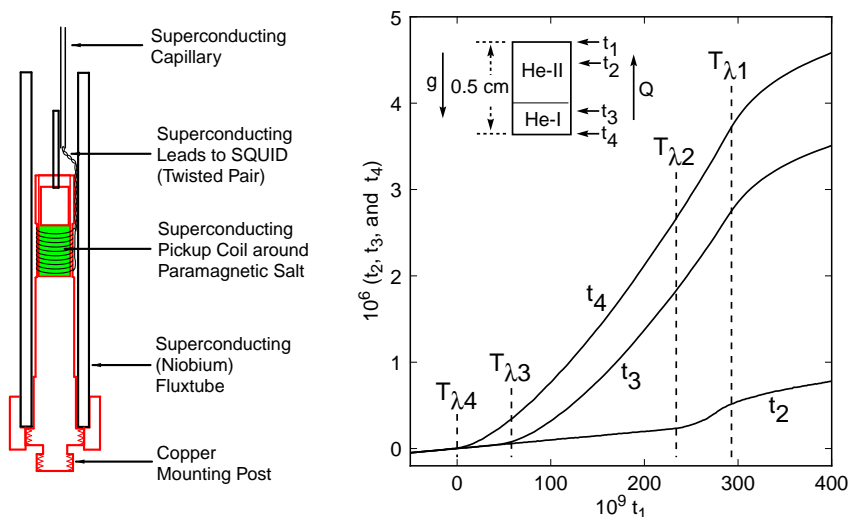


Fig. 2. The left part is a schematic drawing of a susceptibility thermometer⁵ used for measuring temperature with a resolution of a part in 10^{10} . The right part shows measurements⁶ done with four such thermometers which illustrate the gravity effect on the system.

Remarkable advances in thermometry have been made during the last decade or two, largely due to the efforts of the Stanford group headed by John Lipa.³ It is interesting to note that this technology development was, in turn, driven by the opportunity to utilize it in a microgravity experiment, namely the “Lambda-Point Experiment” (LPE) which was carried out on the shuttle Columbia in November 1992.⁴ The thermometer uses supercon-

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ducting technology to measure the magnetization of a paramagnetic salt in a magnetic field. A schematic drawing of a miniaturized version developed in our laboratory⁵ is shown in Fig. 2. A superconducting pickup coil surrounds the salt $[Cu(NH_4)_2Br_4 * 2H_2O]$ and is connected to the input of a SQUID. The salt and coil are surrounded by a superconducting niobium tube which has a magnetic field trapped within it. As the temperature changes, the susceptibility of the salt, and thus its magnetization in the presence of the magnetic field, changes. The change in magnetization leads to a change in the current in the pickup coil which is sensed by the SQUID. In its optimal configuration such a device has been shown to have a resolution $\delta T/T$ in the 10^{-10} to 10^{-11} range, which is determined by thermal ($k_B T$) noise and its finite size (of order 10^{22} particles).¹

The right portion of Fig. 2 shows a set of measurements⁶ on a sample of height $h = 0.5$ cm which illustrates the gravity effect on the transition. Four thermometers are mounted on the sample at the positions indicated schematically by the four arrows in the insert of the figure. They measure the temperatures $t_i \equiv (T_i - T_{\lambda 4})/T_\lambda$, where the $T_i, i = 1, 2, 3, 4$ are the four temperatures and $T_{\lambda 4}$ is T_λ at the sample bottom. A heat current $Q = 80$ nW/cm² passes through the sample from the bottom. The temperature t_1 at the sample top is slowly increased, at a rate of about one nK/sec, from below to above the superfluid transition. Looking back at the right part of Fig. 1, we see that the transition should occur first at the sample bottom. This is indeed the case. At the very left of Fig. 2 the sample is isothermal even in the presence of the heat current because He-II is a virtually perfect heat conductor. All four thermometers yield the same reading. Starting at the point marked $T_{\lambda 4}$, the thermometer at the sample bottom indicates a relatively rapidly rising temperature t_4 , showing that He-I with a thermal gradient across it has formed at the bottom. At the point labeled $T_{\lambda 3}$ the interface between He-I and He-II reached the bottom side thermometer, and t_3 began to rise more quickly. At $T_{\lambda 2}$ the interface passed the top side thermometer, and at $T_{\lambda 1}$ the interface left the sample top and the entire sample finally consisted of He-I. The data show that the difference between $T_{\lambda 1}$ and $T_{\lambda 4}$ is slightly less than $3 \times 10^{-7} T_\lambda$, which agrees quantitatively with the gravity effect expected from Fig. 1.

3. Critical phenomena

The theoretical foundation of critical phenomena is provided by the renormalization-group (RG) theory. The RG concept has opened up the possibility of solving a variety of problems which originate from physical phenomena where infinitely many degrees of freedom interact in an essential

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way. Applications of the RG theory range from elementary particle physics over statistical physics to condensed matter physics. Therefore, successful quantitative tests of the predictions of the RG theory by a study of critical phenomena have implications which transcend the physics of critical points.

The theory of critical phenomena can be tested by experiment at several levels. On the one hand, there are the specific predictions of the RG theory, such as exponent values, which can be examined. In this case, however, the theoretical predictions involve approximate computational methods which yield only approximate results, and thus agreement with experiment is only expected at some approximate level. On the other hand, there are more general concepts such as Universality and Scaling which can be tested. These latter results of the theory are particularly important because these concepts are expected to be *exactly* obeyed by a physical system. Thus highly quantitative experimental results are very desirable.

The “Lambda Point Experiment” was carried out in 1992 by Lipa and coworkers. It yielded measurements of the heat capacity at constant pressure (actually at saturated vapor pressure) much closer to T_λ than is possible on Earth.⁴ The experiment had a number of important implications for our understanding of critical points and for technology development. Here I list some of them very briefly.

1. It showed that phase transitions are “sharp” even for $t < 10^{-9}$.
2. It confirmed the validity of extrapolations towards T_λ of ground-based measurements much further away from T_λ , using theory.
3. It yielded a more precise value of the specific-heat exponent α for comparison with theory.
4. It permitted a more stringent test of the hyperscaling relation $3\nu = 2 - \alpha$ (here ν is the exponent of the correlation length).
5. It led to the development of high-resolution thermometers (HRT’s).
6. It provided important experience with the impact of radiation on HRT’s.
7. It adapted SQUID-based technology to the space environment.

From a science point of view item 4 is perhaps particularly noteworthy. Since the scaling law $3\nu = 2 - \alpha$ is predicted to be exact, any significant deviation from it, no matter how small, would invalidate the applicability of the fundamental structure of the theory to the physical system. The LPE yielded the value $\alpha = -0.0128 \pm 0.0004$. Existing ground-based measurements⁷ gave $\nu = 0.6705 \pm 0.0006$, corresponding to $\alpha = 2 - 3\nu = -0.0115 \pm 0.0018$, in very good agreement with the direct measurement of the LPE. Clearly an even more stringent test of scaling will be possible when ν has been determined more accurately, hopefully within a few years by another microgravity experiment known as the “Superfluid Universality Experiment” (SUE). From a technology viewpoint, we can expect items 5 to 7 to have significant con-

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sequences for numerous future experiments and activities in space as well as on Earth.

A number of other measurements pertinent to critical phenomena should be made, and some of these are already in the planning stages. The experiment SUE is being designed to measure the second-sound velocity and to infer from it the superfluid density along isobars at various pressures near T_λ . The superfluid density gives information about the correlation length, and yields the exponent ν already mentioned above. Exponents like ν should be “universal”, that is they should not change as irrelevant parameters like the pressure are changed. This test of universality at a highly quantitative level again is extremely important because even a small deviation from it would indicate a breakdown of the applicability of the theory. Equally interesting are the non-asymptotic and non-universal deviations from the leading powerlaw behavior of the superfluid density, particularly since a major effort to calculate them with high accuracy is now under way as part of SUE by Dohm and coworkers.

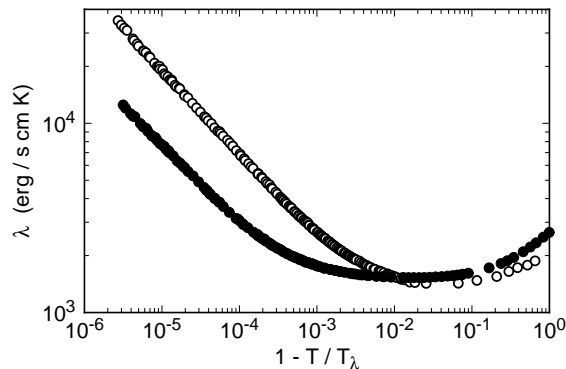


Fig. 3. Thermal conductivity of He-I at saturated vapor pressure (open circles) and at $P = 29$ bar (solid circles). Adapted from Ref. 9.

An important area for high-resolution and microgravity work is that of transport properties. The thermal conductivity of He-I is expected to diverge at T_λ in a manner which is predicted in great detail⁸ by the dynamic RG theory. It has been measured along various isobars⁹ to within a few parts in 10^6 or so of T_λ , and some results are shown in Fig. 3. Modern thermometry permits an extension of the temperature range by about one decade closer to the transition; but then gravity will set the limit. An additional two decades can be gained in microgravity. This is of great importance for experimental tests of theories of *dynamic* critical phenomena,⁸ a relatively unexplored field.

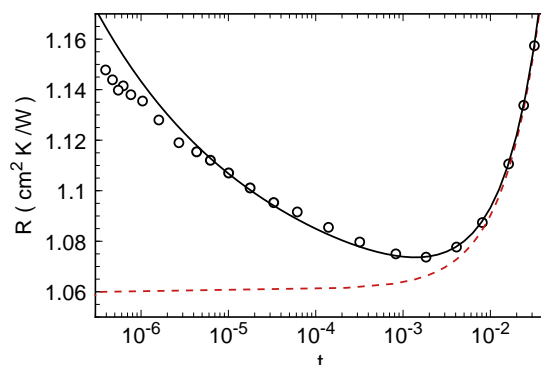


Fig. 4. Thermal boundary resistance R_b at saturated vapor pressure and below T_λ . After Ref. 13.

4. Solid-fluid interfaces and finite-size effects

An important problem is the nature of the boundary conditions which pertain to the interface between a solid surface and a fluid. Generally, these are difficult to study because they influence the fluid properties only over a thickness of order atomic dimensions. This is changed near a phase transition. There is a correlation length over which the properties of the system vary only slowly in space, and this length diverges as the transition temperature is approached. Thus, the boundary layer adjacent to the solid surface acquires a macroscopic thickness. In this boundary layer the thermal conductivity is depressed, and thus there is a singular contribution to the thermal boundary resistance.¹⁰⁻¹² Recent experimental results¹³ are shown in Fig. 4. It is interesting to note that a microgravity environment is not essential for a detailed measurement of this property because the effect is intrinsically confined to a small vertical thickness of a horizontal surface. Nonetheless, quantitative measurements became possible only after the development of very-high resolution thermometry in response to the challenge of specific-heat measurements in microgravity. The results in Fig. 4 are in quite good, but not perfect, agreement with the solid line in the figure which represents the theoretical calculations.¹⁴ The small differences at small ϵ are yet to be explained. A particularly interesting future project will be the measurement of R_b along isobars at elevated pressure. The theory predicts a significant pressure dependence, but so far there are no data.

If the fluid phase adjacent to a boundary is of limited spatial extent, then the boundary will influence significantly also the average macroscopic thermodynamic and transport properties of the system. A critical fluid system which is confined can then be used to study boundary effects. These

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effects can take several forms, depending on the nature of the confinement. For instance, the sample might be contained between parallel plates. On the other hand, different results at the quantitative level are expected when the fluid is contained in long, narrow cylinders. Again the RG theory is expected to provide a unifying framework for understanding these phenomena.

An important question is the validity of finite-size scaling for thermodynamic properties which is expected to hold on the basis of general arguments¹⁵ as well as on the basis of the RG theory.¹⁶ One expects that the properties of the finite system are functions only of the ratio between the characteristic size L of the geometry and the correlation length ξ . In order to test this theory, measurements over the widest possible range of L are necessary. For very small L it is difficult to construct samples with a precisely known and uniform L , and the finite-size effects are more difficult to interpret because they extend beyond the critical region. Nonetheless very valuable measurements have already been carried out on Earth,¹⁷ and additional ones are highly desirable. However, the data for relatively large spacings, up to 10 or 100 μm , which are necessary for a decisive test, are difficult to obtain on the ground. For large L the finite-size effects become apparent only very close to T_λ , and thus are usually obscured by gravity.

A microgravity experiment intended to explore boundary and finite-size effects is the "Confined Helium Experiment" (CHeX) which was launched in November 1997. In it, Lipa and coworkers measured the heat capacity of liquid helium confined between parallel plates with a spacing of 50 μm between them. We are looking forward to the definitive analysis and publication of these results. Similar experiments should be carried out for helium confined in cylindrical tubes, and this would be a logical follow-up on the original CHeX experiment. The cylindrical and parallel-plate geometries are qualitatively different from each other; in the cylindrical case there is no phase transition and only a rounded maximum in the specific heat will occur. In the parallel-plate case the bulk transition is changed to the Kosterlitz-Thouless type.

A particularly interesting and largely unexplored issue is the finite-size effect on transport properties. Very little is known experimentally about transport in confined systems. The thermal conductivity was measured for helium contained in a cylindrical geometry with a 2 μm diameter.¹⁸ Results are shown in Fig. 5. One can scale these data to other cylinder diameters if one extends the finite-size scaling assumption to transport properties. However, one must keep in mind that at this time there is *no* experimental support for this phenomenological theory of transport properties because there exist data only for a *single* cylinder diameter. These data and finite-size scaling yield the results shown in the middle and right parts of Fig. 5

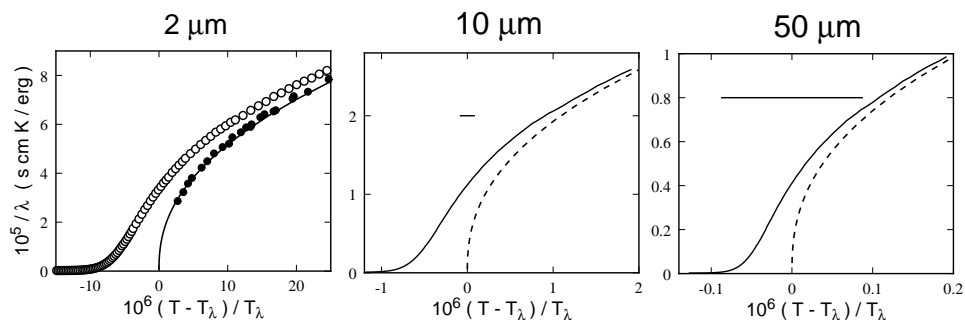


Fig. 5. Left: Thermal resistivity of liquid helium in cylindrical pores of $2 \mu\text{m}$ diameter. Adapted from Ref. 18. Middle and right: The thermal resistivity expected on the basis of the $2 \mu\text{m}$ measurements and finite-size scaling in cylindrical samples of 10 and $50 \mu\text{m}$ diameter. The horizontal bars show the width of the two-phase region due to gravity for a sample of height 3 mm .

for 10 and $50 \mu\text{m}$ diameters respectively. Also shown there is the width of the two-phase region on Earth for a 3 mm high sample. Measurements for the $10 \mu\text{m}$ geometry can perhaps be done on Earth; but for the $50 \mu\text{m}$ sample the finite-size effect would be totally obscured by the gravity effect. It will be very important also for this property to acquire data over a wide pressure range along the λ -line. In principle it should be possible to carry out renormalization-group calculations of the finite-size effect at all pressures without additional adjustable parameters, using the known behavior of the bulk system. However, at present these calculations are not yet available. In the absence of such detailed calculations, the validity of finite-size scaling can still be tested as a function of pressure because the pressure dependence of the correlation length is known from superfluid-density measurements.^{19,20} It will be equally important to carry out measurements over a wide range of cylinder diameters on the ground, and then to extend the size range to its ultimate limit by microgravity experiments. Equivalent experiments should be performed also in a parallel-plate geometry (with Q parallel to the plates) because the two geometries presumably will have different finite-size scaling-functions. Thus there is much work left to be done.

5. Non-equilibrium phenomena

The environment in which we live consists of non-equilibrium systems which are constantly evolving. And yet, much of physics is studied by considering the idealized case of equilibrium. The statistical dynamics of many-particle systems in meta-stable equilibrium or in non-equilibrium is

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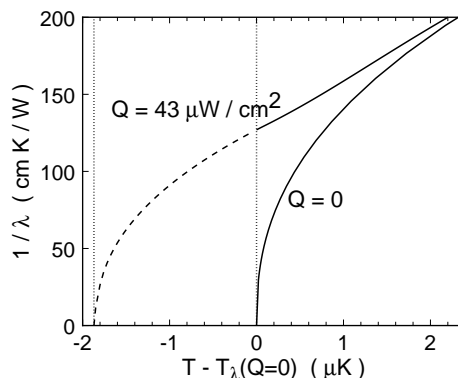


Fig. 6. The thermal conductivity of liquid helium which is expected²¹ in the presence of a finite heat current. Adapted from Ref. 22.

an important field of physics which is far from being well developed and well understood. When non-equilibrium phenomena have been considered, this usually has been done within the framework of linear response theory which applies when departures from equilibrium are small. Indeed in laboratory systems it often is difficult to achieve sufficiently extreme conditions for linear response theory (Ohm's law, Fourier's law, etc.) to break down under well controlled circumstances which permit quantitative study. However, such conditions can be achieved more readily in the vicinity of critical points where typical relaxation times become large. Thus, we can exploit the criticality of a system to explore extreme non-equilibrium conditions where transport becomes a nonlinear phenomenon. The superfluid transition of ⁴He is a system which is particularly suitable for the study of these phenomena. Although much theoretical work has been done on this topic particularly by Haussmann and Dohm,²¹ our understanding of the non-equilibrium system is far from complete. Figure 6 shows the thermal resistance of He-I for $Q = 0$, as well as the theoretically expected²¹ resistance in a finite current of $43 \mu\text{W}/\text{cm}^2$.²² On the basis of the theory a significant current dependence of the resistance is expected; but definitive measurements²³ are only just now becoming available. These effects are best studied at rather small currents, and then again they will be noticeable only very close to T_λ where gravity becomes important. The microgravity experiment DYNAMX is designed to investigate these and other non-equilibrium properties near T_λ . According to present plans it will be carried out on the International Space Station in 2003. However, there are additional aspects which are beyond the scope of DYNAMX which should be pursued, including for instance the effect of non-equilibrium conditions on the thermodynamic properties and upon the behavior of confined systems.

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6. Summary

We have seen that the superfluid transition is unique among phase transitions. The detail in which it can be studied on Earth is limited only by gravity. In microgravity the remaining limitation eventually will be the fluctuations which on the basis of statistical mechanics occur in any finite object at a finite temperature. This system lends itself to the study of diverse physical issues, and the unique opportunities which it has to offer should be utilized by a great variety of experiments in a microgravity environment as well as on the ground. In most cases it is equally important to pursue a vigorous ground-based research program to complement the microgravity measurements. This is the case for instance in the study of finite-size scaling, where the relatively smaller geometries are as essential as the larger ones; the smaller sizes can be studied in Earth's gravity because the finite-size effect is spread out over a relatively wide temperature range (see for instance Fig. 5).

Problems that can be fruitfully examined near T_λ can be classified as belonging to

- a.) critical phenomena *per se*,
- b.) the nature of the boundary between liquids and solids,
- c.) and the breakdown of linear response theory in non-equilibrium systems.

In each of these areas, new knowledge will have a profound impact not only on the particular topic under investigation but also on a broad range of physics. In each of these categories a number of experiments can be fruitfully carried out. Thus there is a large number of exciting high-resolution microgravity and Earth-based experiments to which we can look forward. These experiments will continue well into the new era of the International Space Station (ISS) and will make good use of the low-temperature facility which is planned for the ISS and of the ground-based laboratories presently supported by NASA.

Finally it is important to re-iterate that equally many opportunities for microgravity research exist in other subfields of low-temperature physics, and more broadly in fundamental physics as a whole.

7. Acknowledgement

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