

Search for the “Ultimate State” in Turbulent Rayleigh-Bénard Convection

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Measurements of the Nusselt number Nu and of temperature variations ΔT_b in the bulk fluid are reported for turbulent Rayleigh-Bénard convection of a cylindrical sample. They cover the Rayleigh-number range $10^9 \leq Ra \leq 3 \times 10^{14}$ using He (Prandtl number $Pr = 0.67$), N_2 ($Pr = 0.72$) and SF_6 ($Pr = 0.79$ to 0.84) at pressures up to 15 bars and near-ambient temperatures. The sample had a height $L = 2.24$ m and diameter $D = 1.12$ m and was located in a new High-Pressure Convection Facility (HPCF) at the Max Planck Institute for Dynamics and Self-Organization in Göttingen, Germany. The data do not show the transition to an “ultimate regime” reported by Chavanne *et al.* and are consistent with the measurements of Niemela *et al.*

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Turbulent convection in a fluid heated from below (Rayleigh-Bénard convection or RBC) [1] plays a major role in numerous natural processes. It occurs in Earth’s outer core [2,3], atmosphere [4,5], and oceans [6,7], and is found in the outer layer of the Sun [8] and in giant planets [9]. The intensity of the driving by the thermal gradients usually is expressed by the dimensionless temperature difference known as the Rayleigh number Ra (to be defined explicitly below). Another important dimensionless parameter is the ratio of viscous dissipation to thermal dissipation known as the Prandtl number Pr . The natural phenomena mentioned above generally involve $Ra \geq 10^{20}$ and a wide range of Pr (see, for instance, Ref. [10]), whereas measurements in Earth-bound laboratories, with a few exceptions to be discussed below, had been limited to $Ra \leq 10^{12}$ [1].

Of particular interest has been the global heat transport by the turbulent system, as expressed by the Nusselt number Nu (the ratio of the effective conductivity λ_{eff} of the convecting system to the conductivity λ of the quiescent fluid). Extrapolations of $Nu(Ra, Pr)$ to the geophysically or astrophysically relevant ranges are in question because the basic physics involved in the turbulent flow is expected to change in the vicinity of some Pr -dependent $Ra = Ra^*(Pr)$. For $Pr \approx 1$, Ra^* is estimated to be near 3×10^{14} , and it is expected to increase with Pr approximately as $Pr^{0.7}$ [11]. Unfortunately, on this important issue two nominally equivalent sets of Nu measurements [12–14] in the parameter ranges where Ra^* might be found disagree with each other. One of them, by a group in Grenoble, France [12,15] (the “Grenoble” data) and shown in Fig. 1 as pluses, was interpreted as evidence for Ra^* at an unexpectedly low value near 10^{11} (this evidence will become more obvious in Fig. 3 below). The other, by a group in Oregon [13,16], (the “Oregon” data) shown in Fig. 1 as stars, did not show any transition all the way up to $Ra \approx 10^{17}$. Both

experiments were done at approximately 5 K using helium near its critical point. Their relationships to Ra^* are somewhat uncertain. Because of the proximity of the critical point, Pr increased significantly as Ra increased beyond about 10^{12} and thus the expected value of Ra^* increased.

Here we report new measurements, made at ambient temperatures using compressed helium (He), nitrogen (N_2), and sulfur hexafluoride (SF_6), over the range $10^9 \leq Ra \leq 3 \times 10^{14}$. They are shown as solid symbols in Fig. 1. Our data fall into the narrow range $0.67 \leq Pr \leq 0.84$ characteristic of classical gases where theoretical estimates [11] give $Ra^* \approx 2 \times 10^{14}$, albeit with a considerable uncertainty. Our measurements do not reveal the anticipated transition to an ultimate regime; they are roughly consistent with the Oregon data but are inconsistent with the Grenoble data. It remains unclear to us what caused the transition revealed by the Grenoble results.

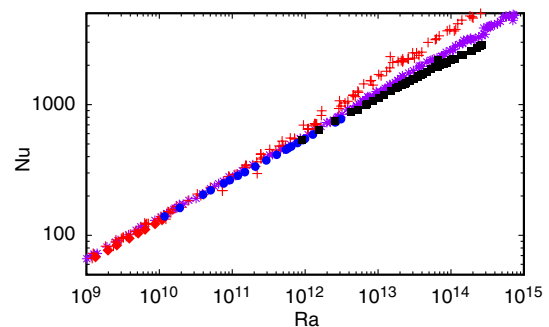


FIG. 1 (color online). The Nusselt number Nu as a function of the Rayleigh number Ra on logarithmic scales. Pluses (red): from Chavanne *et al.* [15]. Stars (purple): from Niemela *et al.* [13] after a reanalysis reported in Ref. [16]. Solid diamonds (red): this work, He. Solid circles (blue): this work, N_2 . Solid squares (black): this work, SF_6 .

For modest, laboratory-accessible, Ra the heat transport is controlled essentially by thin thermal boundary layers (BLs) just above (below) the bottom (top) plate of the convection cell. For $Ra > Ra^*$ it is expected that a large-scale circulation (LSC) in the sample interior will apply sufficient shear to the BLs to cause them to undergo a turbulent transition. This transition was predicted to occur when the shear Reynolds number Re_s , based on the BL thickness, exceeds a critical value $Re_s^* \approx 420$ [11]. It is expected to lead to a new turbulent convective state in the sample interior that was analyzed theoretically by Kraichnan [17] and that is often referred to as the “ultimate state” because it should prevail asymptotically up to infinite Ra . Clearly the experimental discovery and study of the ultimate state is one of the major challenges to experiments in this field, both because of its fundamental interest and because of its geophysical or astrophysical relevance. This is the reason why understanding the difference between the Oregon and the Grenoble data is a major issue in the field of turbulent convection.

Before proceeding to a discussion of our results, it is useful to reexamine the prior large- Ra measurements in more detail. Over 30 years ago it was recognized that the study of RBC using low-temperature helium has unique advantages because of the unusual properties of this fluid [18–20]. This was exploited in early experiments by a group in Chicago who reached $Ra \approx 6 \times 10^{12}$ [21] using a cylindrical sample of aspect ratio $\Gamma \equiv D/L = 0.5$ (D is the diameter and L the height) by approaching the critical region of Helium where Ra tends to become large, but that work did not reveal any transition in the heat transport. Later the Grenoble group [22] carried out similar measurements, also using a cylinder with $\Gamma = 0.5$, and tentatively identified a transition at $Ra^* \approx 10^{11}$. Below that point Nu roughly followed an effective power law $Nu = Ra^{\gamma_{\text{eff}}}$, with $\gamma_{\text{eff}} \approx 0.3$. Above Ra^* , γ_{eff} approached values near 0.4 and was heading toward the asymptotic value 0.5 predicted by Kraichnan. Soon thereafter the same group published a Physical Review Letter entitled “Observation of the Ultimate Regime in Rayleigh-Bénard Convection” [12]. A number of other papers by this group followed with more detail and data up to $Ra \approx 10^{15}$, and we show the results reported in Ref. [15] in Fig. 1 as pluses. We note that Ref. [15] also reports measurements of the vertical flow velocity at half height and half way between the sample wall and center line that yielded global Reynolds numbers consistent with the existence of a LSC up to Ra near 10^{15} . It remained unexplained why the Chicago group did not find the transition seen by the Grenoble group.

A few years later the Oregon group extended the helium measurements to theretofore unprecedented values of Ra as large as 10^{17} by building a cylindrical convection cell with a height $L = 1.0$ m and a diameter $D = 0.5$ m and operating near the critical point of helium [13]. Their data [16] are shown as stars in Fig. 1 and do not reveal any transition. They can be described over the wide range of Ra

by a power law with $\gamma_{\text{eff}} \approx 0.32$ with no significant tendency for γ_{eff} to increase with Ra .

We note that the Chicago, Grenoble, and Oregon experiments were conducted at cryogenic temperatures where experimental difficulties are quite severe and where on average, Pr tends to increase with Ra . It was very desirable to carry out an experiment at near-ambient temperatures using more conventional, and above all different, experimental techniques and fluids with essentially constant Pr that could also achieve the large- Ra values deemed necessary to reach Ra^* . To satisfy those criteria, we used a very large pressure vessel at the Max Planck Institute for Dynamics and Self-Organization in Göttingen, Germany. It is a cylinder of diameter 2.5 m and length 5.5 m, with its axis horizontal, and with a turret above it that extends the height to 4 m over a diameter of 1.5 m. Because of its suggestive shape, this vessel has become known as the “Uboot of Göttingen”. It has an approximate volume of 25 m^3 and can be used up to pressures of 15 bars. In the section containing the turret we placed a RBC sample-cell with $L = 2.24$ m and $D = 1.12$ m (the High Pressure Convection Facility or HPCF), yielding $\Gamma = 0.500$. It had top and bottom plates made of Aluminum and a Plexiglas side wall of thickness 9 mm. The top plate extended from the top a short distance into the side wall; it was water cooled with flow through quadruple spiral channels. The bottom plate was a composite consisting of two aluminum plates with a Plexiglas plate of thickness 5 mm between them, all glued together with Stycast 1266 epoxy. The composite extended from the bottom into the side wall. Joule heating was applied with a heater that was uniformly distributed over the bottom surface of the bottom plate. Aside from that total heat current Q , the heat current Q_s entering the sample could be inferred from the temperature difference across the composite [23,24]. Under most conditions Q_s was equal to Q , indicating no significant parasitic heat losses from the bottom-plate heater. In all cases Q_s was used to compute Nu . Various thermal shields, regulated at appropriate temperatures to prevent heat losses, had been installed as described for instance in Ref. [25]. All spaces outside of the sample and up to a diameter of 1.4 m were occupied by low-density open-pore foam. Estimates indicate that side-wall heat losses were negligible. Each data point was derived from a time series of temperature readings at time intervals of about 5 sec, with each series spanning about 1 d. Based on measurements of thermal relaxation times in the range from 4000 to 10 000 seconds of the top layer of the bottom-plate composite, the first half of each series was discarded to avoid transients and the remainder was time-averaged before computing Ra and Nu . The data were corrected for the finite plate conductivity [25,26]; this correction generally was less than a few percent.

The Rayleigh number is given by $Ra = \beta g L^3 \Delta T / (\kappa \nu)$, and the Prandtl number is $Pr = \nu / \kappa$. Here the thermal expansion coefficient β , the thermal diffusivity κ , and the kinematic viscosity ν were evaluated at the mean

temperature $T_m = (T_t + T_b)/2$ (T_t and T_b are the temperatures at the top and bottom of the fluid, respectively). The gravitational acceleration is g , and $\Delta T = T_b - T_t$. The Nusselt number is $Nu = Q_s L / (A \Delta T \lambda)$ where A is the sample cross-sectional area and λ is the thermal conductivity. The fluid properties were compiled previously [27] from numerous papers in the literature.

Measurements of $Nu(Ra)$ were made with He ($Pr = 0.67$), N_2 ($Pr = 0.72$), and two separate gas fillings (run 1 and run 2) of SF_6 ($Pr = 0.79$ to 84). The data span the ranges $10^9 \leq Ra \leq 3 \times 10^{14}$. The results are shown in Fig. 1 as solid symbols. Figure 2 shows the points in the Ra - Pr parameter space where the various data were taken. Even though at large Ra the Prandtl number is much lower (and thus Ra^* is much lower) than it is for the Oregon or Grenoble data, our data in Fig. 1 reveal no transition to an ultimate regime. The dotted line in Fig. 2 is an estimate of $Ra^*(Pr)$ for cylindrical samples with $\Gamma = 1$ [11] which may serve as a rough guide of where to expect the transition also for $\Gamma = 0.5$. A similar result can be obtained as well by starting with the global Reynolds numbers for $\Gamma = 0.5$ given in Ref. [15]. We see that the Oregon data and our new results exceed that estimate by about the same amount even though the Oregon data extend to much larger Ra . The Grenoble data, on the other hand, do not come near the estimate; thus it seems unlikely that the transition near $Ra = 10^{11}$ which they reveal is associated with a shear-induced turbulence transition in the BLs.

In Fig. 3 we show the data for Nu in more detail by dividing out the approximate Ra dependence $Ra^{0.3}$. For $Ra \leq 10^{11}$ there is general consistency of the dependence of Nu on Ra between all three data sets. Our He data are about 6% below the Oregon and Grenoble data and about 4% below our own N_2 data. Possible causes for these differences may be systematic errors in the calibration of the thermal resistance of the bottom-plate composite (which we estimate to be no larger than 2%), systematic errors in the thermodynamic and transport properties used to compute Ra and Nu for the various fluids which are difficult to estimate but may also be several percent especially for helium near the critical point, as well as an

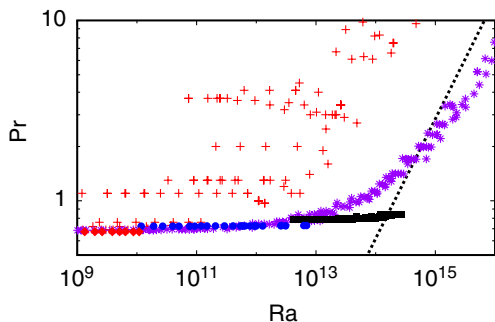


FIG. 2 (color online). The Prandtl number as a function of the Rayleigh number corresponding to the various data sets. The symbols are as in Fig. 1. The dotted line is the estimate of $Pr(Ra^*)$ from Ref. [11] for $\Gamma = 1$.

expected increase of Nu with Pr at constant Ra for $Pr \leq 1$ which also is expected to be a percent or two. For this “reduced” Nusselt number, the break in the Grenoble data at $Ra \approx 10^{11}$ is more apparent than it was in Fig. 1. Clearly this transition to a significantly larger effective exponent, near $\gamma_{\text{eff}} = 0.39$, is not present in the Oregon data, nor can it be found in the Göttingen data. In general our data are slightly lower than the Oregon data, and this difference increases as Ra increases. In part this may again be due to systematic errors in the fluid properties that were used in the analyses, but at the larger Ra a dependence of the Oregon Nu data on Pr may also be contributing. As illustrated in Fig. 2, Pr starts to increase for the Oregon data as Ra exceeds about 10^{12} , whereas for our data Pr is nearly independent of Ra .

A notable feature of the Göttingen data is a significant change of the dependence of Nu upon Ra near $Ra = 6 \times 10^{13}$. Hardly noticeable in Fig. 1, it becomes apparent with the much higher resolution of Fig. 3. We do not know the cause of this, but expect that it is unrelated to the ultimate-state transition since it leads to a *decrease* from about 0.308 to about 0.297 of the effective exponent γ_{eff} rather than an increase toward 0.39 as found by the Grenoble group for $Ra \approx 10^{11}$.

An interesting and instructive feature of this system is the vertical temperature drop ΔT_b across the bulk of the fluid [28]. Whereas the temperature drop ΔT_{BL} across the two boundary layers dominates $\Delta T = \Delta T_{\text{BL}} + \Delta T_b$, values of order 0.1 have been found for $\Delta T_b / \Delta T$ in experiments with water, Ra near 10^9 , and near the side wall where plumes travel from one plate to the other (any temperature drop along the sample axis where plumes are scarce was found to be considerably smaller). In the range $Ra < Ra^*$ the gradient in the bulk is expected to remain small, but as Ra^* is exceeded, the BLs are expected to become turbulent

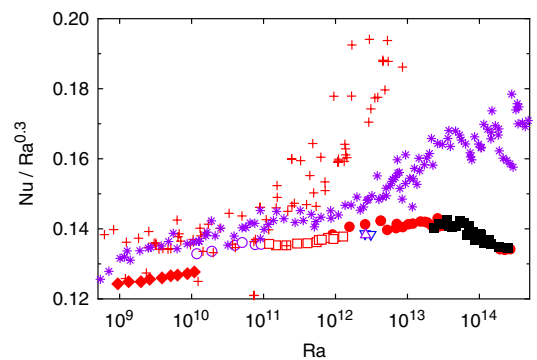


FIG. 3 (color online). The reduced Nusselt number $Nu/Ra^{0.3}$ as a function of the Rayleigh number Ra . Pluses (red): from Chavanne *et al.* [15]. Stars (purple): from Niemela *et al.* [13] after a reanalysis reported in Ref. [16]. Solid diamonds (red): this work, He, $P = 4.3$ bars. Solid squares (black): SF_6 , run 1. Solid circles (red): SF_6 , run 2. Open symbols: This work, N_2 . Circles (purple): $P = 2$ bars. Squares (red): $P = 6$ bars. Triangles pointing down (blue): $P = 10$ bars. Triangle pointing up (purple): $P = 15$ bars.

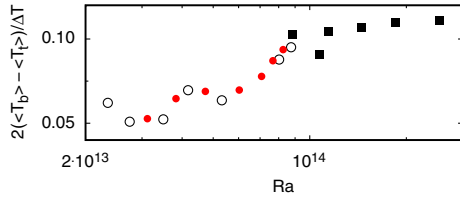


FIG. 4 (color online). The temperature drop $2(\langle T_b \rangle - \langle T_l \rangle)/\Delta T$ in the bulk near the side wall. Open (solid) circles: $P \approx 8$ bars, decreasing (increasing) pressure. Solid squares: $15 \approx P \approx 9$ bars.

and their thermal resistance presumably is reduced. Then a much larger part of the temperature drop should occur across the bulk. With this in mind, the azimuthal and temporal mean temperatures $\langle T_b \rangle$ and $\langle T_l \rangle$ along the side wall at the vertical positions $-L/4$ and $L/4$, respectively, were determined with thermometers imbedded in the side walls as described for instance in Ref. [28]. Those data yield the estimates $\Delta T_b \approx 2(\langle T_b \rangle - \langle T_l \rangle)$ shown in Fig. 4. There is some potentially interesting structure in the Ra dependence of this quantity, but the overall value of $\Delta T_b/\Delta T$ remains modest at all Ra and is comparable to similar results reported elsewhere [28]. The results imply that $\Delta T_{BL} \gg \Delta T_b$, consistent with $Ra < Ra^*$, for our entire parameter range.

In this Letter we report new measurements of the Nusselt number, and of the bulk temperature gradient near the side wall, for turbulent Rayleigh-Bénard convection in a cylindrical sample of aspect ratio 0.5 over the Rayleigh-number range $10^9 \lesssim Ra \lesssim 3 \times 10^{14}$ and for Prandtl numbers close to 0.8. The results do not reveal any evidence for a transition to the “ultimate” regime expected in the large-Ra limit and thus indicate that $Ra^* > 3 \times 10^{14}$ for $Pr \approx 0.8$. We note that theoretical estimates predict that $Ra^* \propto Pr^\alpha$ with $\alpha \approx 0.7$. Thus, for the Grenoble and the Oregon measurements, which involved larger values of Pr, any transition should be expected only at even larger values of Ra^* . In view of these conclusions we believe that the transition reported by the Grenoble group near $Ra = 10^{11}$ is unrelated to the transition to the ultimate regime and that its origin must be sought elsewhere.

An interesting issue that remains for the future is whether the ultimate regime exists at any Ra for a RBC system with rigid top and bottom plates. We cannot rule out that the LSC disintegrates with increasing Ra into fluctuations of smaller scale, and that the fluctuation intensity simply stays below the level that would lead to a transition.

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