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Scaling and universality of the thermal conductivity of liquid ^{4}He near the superfluid transition and in restricted geometries

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Abstract

We present measurements of the thermal conductivity $\lambda(t,P,L) = 1/\rho(t,P,L)$ near the superfluid transition of ^{4}He at saturated vapor pressure and confined in cylindrical geometries with radii $L = 0.5$ and 1.0 μm ($t \equiv T/T_\lambda(P) - 1$). For $L = 1.0$ μm measurements at six pressures P are presented. At and above T_λ the data are consistent with a universal scaling function $F(X) = (L/\xi_o)^{x/\nu}(\rho/\rho_0)$, $X = (L/\xi_o)^{1/\nu}t$ valid for all P (ρ_0 and x are the pressure-dependent amplitude and effective exponent of the bulk resistivity $\rho(t,P,\infty) = \rho_0 t^x$ and $\xi = \xi_0 t^{-\nu}$ is the correlation length). Indications of breakdown of scaling and universality are observed below T_λ.

Keywords: Thermal conductivity of liquid ^{4}He; Superfluid transition; Liquid ^{4}He

More than a quarter century ago William Fairbank at Stanford University had a vision of making heat capacity measurements near the superfluid transition of ^{4}He with an unprecedented resolution which could only be reached in a microgravity environment. One motivating force behind this endeavor was a simple question: are the changes in the physical properties of materials at continuous phase transitions (i.e. “critical points”) really “sharp”, or are they smoothed out over a finite temperature range, albeit perhaps a very small one. Ten years ago measurements by John Lipa et al. (1996) in the microgravity environment of the space shuttle Columbia not only answered the original question with the “Lambda Point Experiment” (LPE), but also opened up a completely new era of very-high resolution experiments. Ultimately, in 1997, this led to the establishment within the National Aeronautics and Space Administration (NASA) of the Discipline for Fundamental Physics (FP). The creation of the FP Discipline opened up large areas of new opportunities for fundamental microgravity research in several subfields of physics, including Low-Temperature and Condensed-Matter Physics, Laser Cooling and Atomic Physics, and Relativity and Gravitation.

Whereas the LPE was aimed at the study of a phase transition under conditions where the confinement of the sample by its container had a negligible influence, we are addressing in this paper just the opposite question, namely how the confining surfaces cause a “rounding” of the transition. Near a phase transition there is a correlation length ξ over which the properties of the system vary only slowly in space, and this length diverges as the transition temperature is approached. Thus, the boundary layer adjacent to a solid surface acquires a macroscopic thickness. If the fluid phase adjacent to a boundary is of limited spatial extent, then the boundary will influence significantly also the average macroscopic thermodynamic and transport properties of the system.

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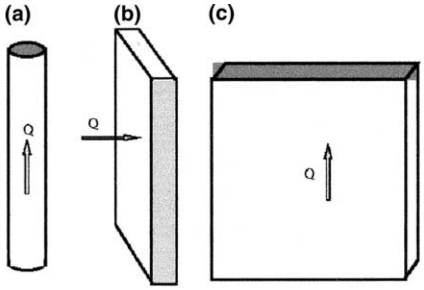

Fig. 1. Three different universality classes for finite size effects of transport properties: (a) one-dimensional with the current along the axis; (b) two-dimensional with the current orthogonal to the plane; (c) two-dimensional with the current in the plane.

As illustrated in Fig. 1, these effects can take several forms, depending on the nature of the confinement. For instance, the sample might be contained between parallel plates (b and c). On the other hand, different results are expected when the fluid is contained in long, narrow cylinders (a). In the case of transport properties, there is also the direction of the current relative to the geometrical axes which may determine the influence of the geometry. Thus, for the thermal conductivity one would expect at least three distinct "universality classes" for the finite size effects as illustrated in Fig. 1.

Phenomenologically one might argue that the rounding of the phase transition can be described by a finite-size scaling theory. In this approach one expects that the properties of the finite system are functions only of the ratio between the characteristic size L of the geometry and the correlation length ξ. In order to test this theory, measurements over the widest possible range of L are necessary. For very small L it is difficult to construct samples with a precisely known and uniform L, and the finite-size effects are more difficult to interpret because they extend beyond the critical region. The data for relatively large spacings, up to 10 or 100 μm, which are necessary for a decisive test, cannot be obtained on Earth because the finite-size effects become apparent only very close to T_λ and thus are obscured by gravity. A microgravity experiment which explored boundary and finite-size effects on thermodynamic properties is the "Confined Helium Experiment" (CHeX) which was flown on Columbia in November 1997. With it, Lipa et al. (2000) measured the heat capacity of liquid helium confined between parallel plates with a spacing of 50 μm between them. In the microgravity experiment "Boundary Effects on the Superfluid Transition" (BEST) we plan to carry out measurements of a transport property, namely of the thermal conductivity, in the microgravity environment of the International Space Station. In support of this mission, we are now making ground-based measurements for the smaller values of L where gravity effects are relatively unimportant because the finite-size effects occur over a larger temperature range. Here we report our early results. Our samples are confined in "microchannel plates" which consist of glass disks penetrated by closely-spaced holes of remarkably uniform shape and size. Typical electron micrographs are shown in Fig. 2. Both cylindrical and rectangular geometries are available, and thus we will be able to investigate both case (a) and case (c) illustrated in Fig. 1. Our present results are only for case (a).

The modern theory of critical phenomena (Fisher, 1998) predicts that continuous phase transitions belong to distinct universality classes which are determined by such general properties of the system as the number of degrees of freedom of the order parameter and the spatial dimensionality. Within a given class, exponents and amplitude ratios are identical (i.e. universal) for all members and independent of irrelevant variables. An example of an irrelevant variable is the pressure P of a liquid helium sample at which measurements near the superfluid transition temperature T_λ are made. Within a given universality class, the dependence of many properties upon certain parameters can be represented by

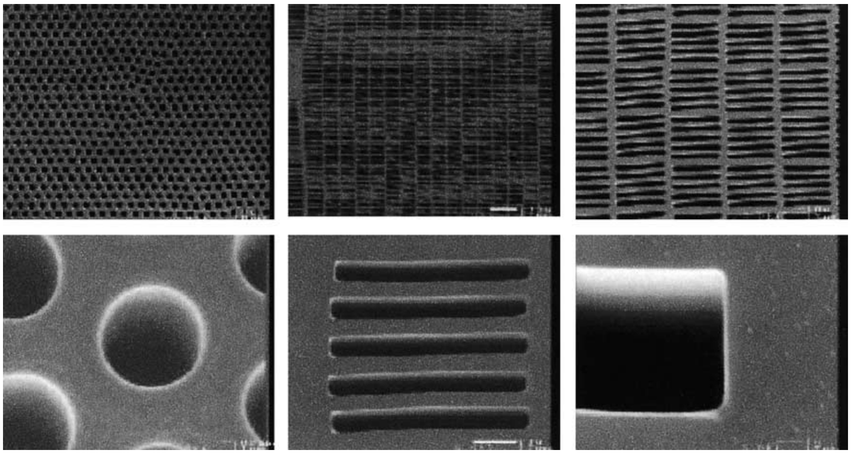

Fig. 2. Electron micrographs of microchannel plates. Left: two magnifications of cylindrical holes of 1 μm diameter in plates of thickness 1 mm. Middle and right: four magnifications of rectangular holes of size 5 × 50 μm in plates of thickness 2 mm.

scaling functions which are the same for all systems. Thus, our work is an experimental study of the scaling function which describes the effect of confinement in a cylindrical geometry with radius L on a transport property near a critical point (Fisher, 1971; Privman, 1990). For static properties this finite-size scaling has been studied by a number of precise experiments. For example, the heat capacity near the superfluid transition of ^{4}He has been measured for confinement sizes which vary by a factor of over 1000, and the data to a large extent can be collapsed upon a unique function when properly reduced (Mehta and Gasparini, 1997; Mehta et al., 1999; Kimball et al., 2000; Lipa et al., 2000). Even for static properties, however, measurements which test the universality of such a scaling function are quite limited (Kimball and Gasparini, 2001, 2002). For transport properties there are, to our knowledge, no prior experiments which test scaling and universality for finite-size effects. There has been only one experiment on the effects of confinement on a transport property (Kahn and Ahlers, 1995; Ahlers, 1999), namely the measurements of the thermal conductivity λ of helium near T_λ in cylindrical tubes. These measurements can be used to derive a scaling function which would be expected to be universal, but since they were performed only for the one value $L = 1$ μm and only at saturated vapor pressure (SVP), they provided a test of neither finite-size scaling nor of the universality of the derived function.

Here we present experimental results for the thermal resistivity $\rho(t,P,L) \equiv 1/\lambda(t,P,L)$ near $T_\lambda(P)$ of liquid ^{4}He confined in cylinders of two different radii and at various pressures as a function of the reduced temperature $t \equiv T/T_\lambda - 1$. The use of two confinement sizes allows us to directly test finite-size scaling, while the use of different pressures for one size provides a test of universality. For bulk helium $\rho(t,P,\infty)$ depends strongly on pressure (Tam and Ahlers, 1985), so that a comparison of an appropriate scaling function for $\rho(t,P,L)$ at different pressures provides a sensitive test of universality. These two aspects were tested in separate experiments: measurements as a function of L were taken at SVP, and measurements as a function of P were taken at a single confinement size $L = 1.0$ μm.

Theoretical predictions for λ are still quite limited. Monte Carlo calculations give the shape of a scaling function, but only to within a multiplicative factor (Nho and Manousakis, 2001). Within its precision this shape agrees well with the measurements (Kahn and Ahlers, 1995). Very recently, a one-loop renormalization group (RG) calculation of $\lambda(t,P,L)$ for $t \geqslant 0$ and at SVP was carried out (Töpler and Dohm, 2003), but at present there are no such calculations for $t < 0$ and for higher pressures. Thus, in order to provide a broader framework for the analysis of our data, we use a phenomenological approach. We assume that the temperature and size dependence of ρ are separable and that the size dependence is a function only of L/ξ where $\xi = \xi_0 t^{-\nu}$ is the correlation length: $\rho(t,P,L) = \rho(t,P,\infty)\tilde{F}(L/\xi)$. Since $\rho(t,P,\infty)$ goes to zero as t does while $\rho(t,P,L)$ remains finite, $\tilde{F}$ diverges at $t = 0$. To avoid this difficulty, we re-define the scaling function as $F(X) = (L/\xi)^{x/\nu}\tilde{F}$ which avoids the divergence at $t = 0$. Consistent with experiment (Tam and Ahlers, 1985), we have written ρ for bulk helium as a power law $\rho(t,P,\infty) = \rho_0 t^x$ with effective exponents $x(P)$ and amplitudes $\rho_0(P)$. We now have

$$F(X) = [L/\xi_0(P)]^{x/\nu}[\rho(t,P,L)/\rho_0(P)] \tag{1}$$

with

$$X = (L/\xi_0)^{1/\nu}t. \tag{2}$$

The correlation length has a pressure-dependent amplitude $\xi_0(P)$ and a universal exponent $\nu = 0.6705 \pm 0.0006$ (Goldner et al., 1993). The values of ξ_0, ρ_0 and x are known from bulk measurements (Tam and Ahlers, 1985) and are summarized in Table 1.

Our results for the resistivity at SVP are plotted versus t in Fig. 3 for two different values of L. The data show the effect of confinement, with the smallest size showing the greatest rounding of the transition and the greatest increase of $\rho(t = 0)$. The scaling variable $F(X)$ (Eq. (1)) is plotted versus X (Eq. (2)) for the two sizes in Fig. 4. Except for $X \lesssim -2$, the data collapse onto a single curve, thus supporting the concept of finite-size scaling. The small difference in F between the two data sets below $X \simeq -2$ suggests a breakdown of finite-size scaling in the superfluid phase.

Fig. 5 shows $\rho(t,P,L)$ as a function of t for six different values of P and $L = 1.0$ μm. Whereas the resistivity of the bulk fluid drops to zero at $t = 0$, (Tam and Ahlers,

Table 1
Parameters relevant to the experimental runs, and values of the scaling function $F(X)$ at $X = 0$

L (μm)	P (bar)	ξ_0 (nm)	$10^{-4}\rho_0$ (W/K)	x	$F(0)$
0.5	SVP	0.1432	8.312	0.4397	1.35
1	SVP	0.1432	8.312	0.4397	1.35
1	SVP	0.1432	8.312	0.4397	1.40
1	6.95	0.1425	9.073	0.4251	1.30
1	11.25	0.1410	10.19	0.4250	1.30
1	14.73	0.1399	11.10	0.4250	1.32
1	22.31	0.1382	12.79	0.4159	1.31
1	28.00	0.1314	15.07	0.4127	1.24

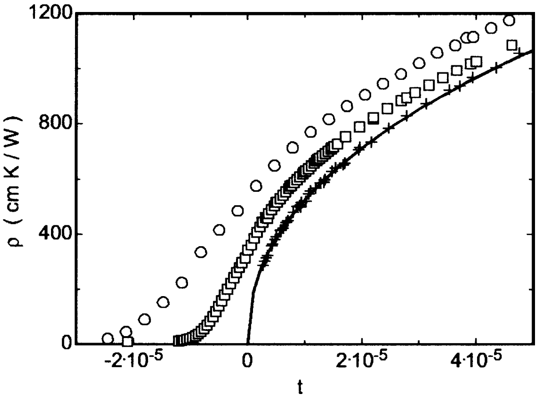

Fig. 3. Thermal resistivity versus reduced temperature at SVP for $L = 0.5$ μm (open circles) and 1.0 μm (open squares). The plusses are bulk measurements (Tam and Ahlers, 1985) and the dashed curve is a powerlaw fit to the bulk data.

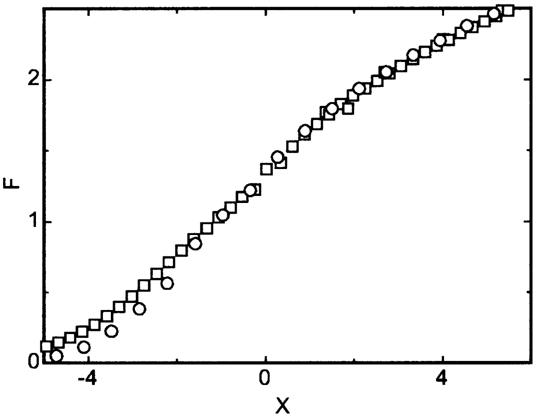

Fig. 4. Scaling function F versus scaling variable X at SVP for $L = 0.5$ μm (open circles) and 1.0 μm (open squares).

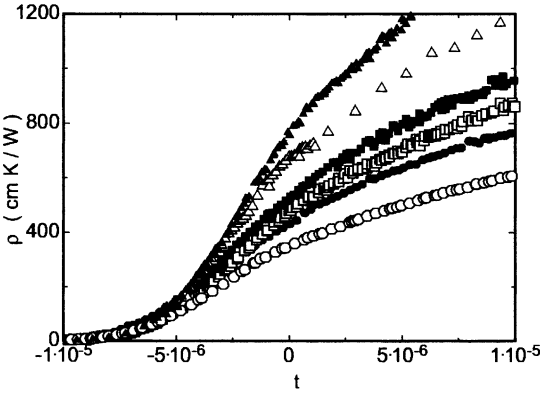

Fig. 5. Thermal resistivity versus reduced temperature for $L = 1.0$ μm at SVP (open circles); 6.95 bars (solid circles); 11.25 bars (open squares); 14.73 bars (solid squares); 22.31 bars (open triangles); 28.00 bars (solid triangles). The reduced temperature for each pressure is defined relative to $T_\lambda(P)$.

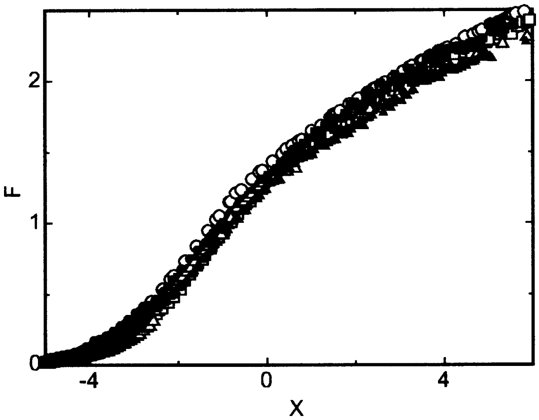

Fig. 6. Scaling function F versus scaling variable X for $L = 1.0$ μm. Pressures and symbols are as in Fig. 5.

1985) that of the finite system remains finite at $t = 0$ and decreases smoothly to very small values as t becomes more negative. The value of $\rho(t = 0, P, L)$ varies by nearly a factor of three for the pressures used.

In Fig. 6, the function F is plotted versus X for six different pressures. Within the resolution of that figure the data collapse on the same curve, suggesting that a single scaling function describes all six pressures. The collapse occurs despite the large variation of ρ at constant t. The values of F at $X = 0$ are given in Table 1. However, closer inspection suggests that universality breaks down below $T_\lambda(P)$. To explore this further, we show in Fig. 7 the scaling function $F(X)$ on a logarithmic scale as a function of X on a linear scale in the range below T_λ. For $X > -2$ one sees that the data collapse within their resolution. However, at more negative X there is a systematic trend of $F(X)$ with P which exceeds the scatter of the data. This is seen from the insert in the figure, which

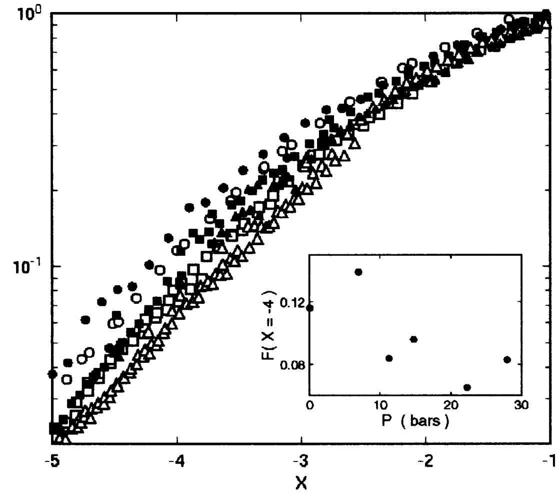

Fig. 7. The scaling function F below T_λ on a logarithmic scale versus X on a linear scale for $L = 1.0$ μm. Pressures and symbols are as in Fig. 5. The insert shows $F(X = -4)$ as a function of the pressure.

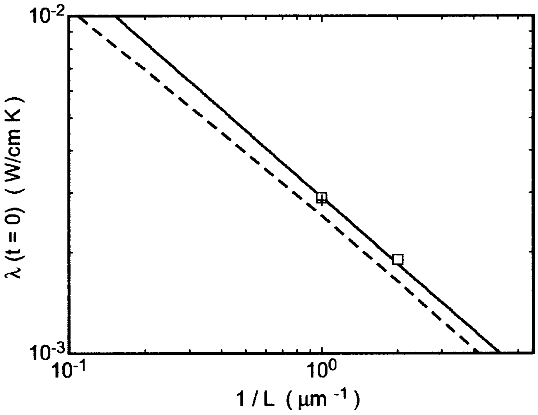

Fig. 8. Thermal conductivity $\lambda(t = 0)$ vs L^{-1} on logarithmic scales. The plus is the SVP measurement from one experimental apparatus, and the open squares were obtained with a second apparatus. The solid straight line is the prediction based on Eq. (1). The dashed curve is the prediction by Töpler and Dohm (2003).

gives F at $X = -4$ as a function of pressure. At $X = -4$ the results for $F(X)$ vary from about 0.13 at small P to about 0.07 at high P.

Aside from testing scaling and universality, an important issue is to what extent detailed theoretical calculations can reproduce the conductivity. As discussed above, the theoretical information is limited. Monte Carlo calculations, which give the shape of the scaling function quite well, involve as yet undetermined parameters. However, the recent renormalization group calculations have yielded results for λ at SVP (Töpler and Dohm, 2003). In Fig. 8, we show data for $\lambda(t = 0)$ as a function of L^{-1} on logarithmic scales. The phenomenological scaling function Eq. (1) predicts $\lambda(t = 0) \propto L^{-x/\nu}$ which, for $x/\nu = 0.656$ is shown by the solid straight line. The RG prediction is given by the dashed line. It falls only about 15% below the data, and in the experimental range of L it has the same effective exponent (the slope of the curve) as the data and the scaling prediction. The excellent agreement with the RG calculation is particularly gratifying since all adjustable parameters in the theory are taken from properties of the infinite system and from static finite size properties (Töpler and Dohm, 2003).

Future plans call for the measurement of the resistivity at larger L, with the largest (50 μm) to be flown on the International Space Station, extending the range covered to two decades in L.

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