

Singularity in the Thermal Boundary Resistance between Superfluid ^4He and a Solid Surface

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We report new measurements in four cells of the thermal boundary resistance R between copper and ^4He below but near the superfluid-transition temperature T_λ . For $10^{-7} \leq t \equiv 1 - T/T_\lambda \leq 10^{-4}$ fits of $R = R_0 t^{-x_b} + R_b$ to the data yielded $x_b \approx 0.18$, whereas a fit to theoretical values based on the renormalization-group theory yielded $x_b = 0.23$. Alternatively, a good fit of the theory to the data could be obtained if the *amplitude* of the prediction was reduced by a factor close to 2. The results raise the question whether the boundary conditions used in the theory should be modified.

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An important, as yet unresolved, issue in condensed-matter physics is the nature of the boundary conditions which are appropriate to describe the interface between a liquid and a solid. Experimentally it is difficult to investigate this problem because the boundary layer usually is of atomic dimensions and contributes negligibly to macroscopic properties averaged over a bulk sample. This situation is changed near a critical point, where the thickness of a boundary layer is of the same order as the correlation length ξ ; the correlation length diverges at the critical temperature. A system particularly suitable for the investigation of the boundary layer is the superfluid transition of ^4He at T_λ . In this Letter we report measurements of the thermal boundary resistance between superfluid ^4He and a solid surface very near T_λ . We find that these measurements cannot be reconciled with theoretical calculations based on the renormalization-group theory (RGT) of critical phenomena [1,2]. One possible explanation would be a fundamental failure of the RGT, but we regard this explanation as unlikely. An alternative explanation may be found in the boundary conditions used in evaluating the theoretical result. The theory assumes that the order parameter vanishes at the solid-liquid interface. We believe that the data imply that this assumption of Dirichlet boundary conditions, which is made in most realistic calculations of finite-size and surface effects near critical points, may have to be reexamined.

In bulk superfluid ^4He heat transport occurs through the counterflow of the normal-fluid (j_n) and superfluid (j_s) currents and does not produce a temperature gradient [3]. However, Landau predicted [3] that a heat current Q orthogonal to a solid wall should produce a boundary layer in the fluid adjacent to the wall with a temperature difference ΔT_b across it. The reason for this is twofold. As recognized by Landau, "the transition of the superfluid liquid into the normal one takes place in a thin layer near the surface." In addition, the suppression of the superfluid density near the wall leads to a reduction of j_s and j_n . The phenomenon is illustrated schemati-

cally in Fig. 1. Within this boundary layer, suppression of the counterflow implies that some of the heat must be carried by diffusive processes. Consequently, a thermal gradient is developed within it. However, the thermal resistance $R_b = \Delta T_b/Q$ is unobservably small deep in the superfluid phase where the boundary-layer thickness is of atomic dimensions. Only very close to the superfluid transition temperature T_λ where $t = 1 - T/T_\lambda$ becomes small and $\xi = \xi_0 t^{-\nu}$ becomes large does R_b become measurable in high-precision experiments. Thus this phenomenon was found experimentally only relatively recently [4–6]. Roughly speaking one can assume that the temperature gradient in the boundary layer decays exponentially from $-Q/\lambda$ at the wall (where there is presumed to be no counterflow at all) to zero deep in the superfluid (where all the heat is carried by counterflow) with a characteristic length equal to ξ , as illustrated in Fig. 1. Here $\lambda \approx \lambda_0 t^{-x_\lambda}$, with an effective exponent $x_\lambda \approx 0.43$, is the diffusive thermal conductivity of the fluid [7]. This leads to the estimate

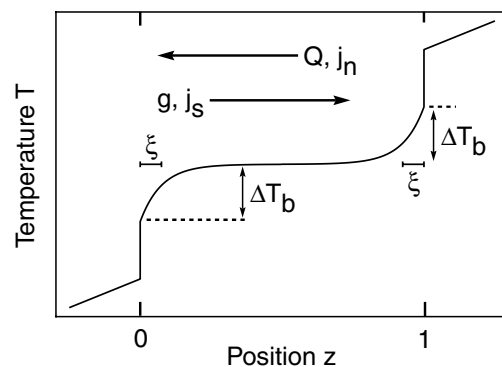


FIG. 1. Schematic diagram (not to scale) of the temperature variation in the cell. The discontinuity at $z = 0$ ($z = 1$) is due to the classical Kapitza resistance R_K at the top (bottom) of the cell, and ΔT_b (which develops over a length scale ξ) is due to R_b . The directions of the currents Q , j_n , j_s , and of gravity g are indicated.

$$R_b \approx \xi/\lambda \propto t^{-x_b}, \quad (1)$$

where $x_b = \nu - x_\lambda \approx 0.24$ [8]. A renormalization-group-theoretical calculation of R_b carried out by Frank and Dohm (FD) [1,2] agrees with this qualitative picture and is expected to give the behavior of $R_b(t)$ quantitatively.

The singular boundary resistance is particularly interesting because it is a direct consequence of the boundary conditions (BC) which are relevant at the solid-liquid interface. For instance, theoretically popular (but physically unrealistic) BC in the study of finite-size effects near critical points are periodic ones (PBC) [9]. For PBC, there is *no* contribution in the fluid to the boundary resistance at the solid-liquid interface because the order parameter is not suppressed. More realistic BC are of the Dirichlet type (DBC). In that case it is assumed that the order parameter *vanishes* at the solid wall. This is the case considered by FD and illustrated in Fig. 1. In principle, one can imagine a continuous family of BC [10], and an important question that goes well beyond the problem of critical phenomena is which BC are relevant to real physical systems.

We find that excellent fits to new data for four separate cells (labeled I to IV) can be obtained with the power-law Eq. (1), but that the resulting values of x_b are close to 0.18 and thus significantly smaller than the theoretically expected value $x_b^{th} \approx 0.24$. A disagreement about the exponent value between theory and experiment would suggest a fundamental problem in the RG treatment of the boundary resistance. However, we were also able to obtain a reasonable fit of the function predicted by the theory [2] to the data when this function was multiplied by a factor significantly less than 1 and, for three of the cells, close to 0.53. The amplitude depends on the boundary conditions which apply. Thus, at this point it is not necessary to invoke a fundamental failure of the theory. Instead, the discrepancy indicates a need to consider modifications of the DBC which have been assumed in the theory not only of R_b , but also of finite-size effects on the specific heat [11,12].

The apparatus was similar to one described previously [5,13,14]. We used four different sample cells. Cell I was described elsewhere [15]. It had a diameter of 1.27 cm. The 0.12 mm thick stainless steel sidewall was brazed to a top copper piece, leaving no gap between it and the copper [16]. The copper surface at the sample top was finely machined and had a measured root-mean-square (rms) roughness of $1 \mu\text{m}$. The fill capillary entered the cell bottom. Two side planes (SPs) similar to those first used by Duncan *et al.* [4,5] were used to measure the helium temperature. They pressed around the entire circumference against the outside surface of the sidewall. Thermometers on the SPs could sense the helium temperature, unencumbered by any temperature drop across a solid-liquid interface. Cells II and III had a similar construction [14]. However, the cell *bottom* was sealed to the *bottom* copper piece with

epoxy (brazing) for cell II (III), and the fill lines entered the cells from the top. The bottom surfaces were polished copper and the rms surface roughness was 0.01 (0.05) μm for cell II (III). Cell IV was similar to one used in the NASA flight project DYNAMX (Critical Dynamics in Microgravity) [17]. It had copper top and bottom cylinders of diameter 2.28 cm which were epoxied into a stainless steel wall of thickness 0.075 mm. The two copper surfaces were polished and had a rms roughness of about 0.02 μm . The wall was *penetrated* by two copper SPs which made contact all around the circumference with the helium. The thickness of each SP was 0.075 mm. The fill capillary entered the cell top.

The temperatures of the top (T_t), bottom (T_b), and the two side planes (T_{st} and T_{sb}) were measured with nK resolution [18]. In each case the fill capillary came from an overflow volume containing the liquid-vapor interface on an isothermal shield which was kept at a temperature near 2.2 K and steady to $\pm 1 \mu\text{K}$ [19]. The surface *not* penetrated by the fill capillary was used for the measurements. The sample was ^4He with less than 1 ppb ^3He .

The calculation of Frank and Dohm [2] gives the difference $\delta R_b^{th} = R_b^{th}(t) - R_b^{th}(t_0)$ and thus the values of R_b^{th} are determined only to within an additive constant. The theoretical result has the form

$$\delta R_b^{th} = \int_{t_0}^t \frac{dt'}{t'} \frac{\xi}{2\lambda} K(f, w, u, \gamma). \quad (2)$$

Here λ and ξ are the thermal conductivity and correlation length above T_λ , and K is a function of the static and dynamic coupling constants γ , u , f , and w of model F [20] for the dynamics of the superfluid transition. The coupling constants depend on t and are known from independent fits of the theory [21] to the thermal conductivity [22] and heat capacity [22,23] near T_λ . Using their tabulated values [24], we evaluated δR_b^{th} numerically from the equations given in Ref. [2] and $t_0 = 0.05$.

In Fig. 2 the data for $R = R_b + R_B$ are shown for all four cells on a linear vertical scale. For cell I there are data for several power densities Q which show that, within the experimental resolution, the results do not depend on Q [16]. There is a large sample-dependent background contribution $R_B = R_K + R_{Cu}$ due to the usual Kapitza resistance R_K and the series resistance R_{Cu} in the copper parts of the cell (see Fig. 1). The theoretical prediction [2] of R_b , after vertical shifts which cause agreement with the data near $t = 10^{-4}$, is given by the solid lines in the figure. Obviously the agreement is not good.

Equation (1) suggests that it should be possible to fit the data near T_λ with a power law. Thus, as an alternative interpretation we considered the data for very small t where we can represent R_B by a constant. We fitted the function

$$R = R_0 t^{-x_b} + R_B \quad (3)$$

to the data with $t < 10^{-4}$, least-squares adjusting R_0 , x_b , and R_B . For $t \equiv 1 - \bar{T}/T_\lambda$ we used the zero-current value

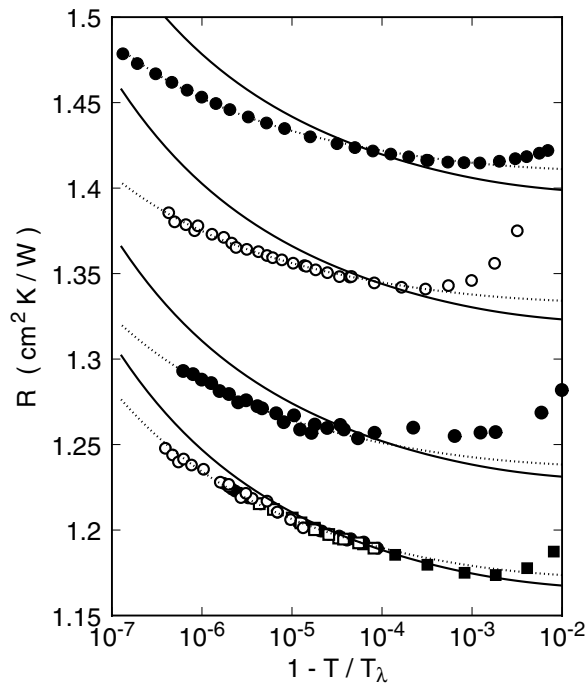


FIG. 2. The resistance R of (from bottom to top) cells I (moved up by $0.1 \text{ cm}^2 \text{ K/W}$), II, III, and IV (moved down by $0.2 \text{ cm}^2 \text{ K/W}$). For cell I the open circles, solid circles, open squares, and solid squares are for data taken at $2, 20, 30,$ and $40 \mu\text{W}/\text{cm}^2$, respectively. The solid lines are the theoretical result, shifted vertically so as to agree with the data near $t = 10^{-4}$. The dotted lines are the theory multiplied by factors of $0.76, 0.59, 0.51,$ and 0.52 for cells I to IV, respectively, and shifted vertically.

of T_λ at the vertical position of the cell surface in question [25] and $\bar{T} \equiv T_{He} + \Delta T_b/2$. We obtained the values of x_b shown in Fig. 3a. The error bars are 95% confidence limits derived from the random errors and do not include

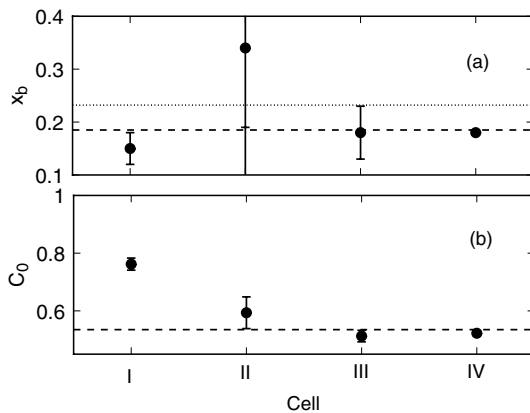


FIG. 3. (a) Values of x_b derived from fitting Eq. (3) to the data for cells I to IV. The dotted line is the theoretical value, and the dashed line is an average value based on the experiments. (b) Values of C_0 derived from fitting Eq. (4) to the data for cells I to IV. The theoretical value corresponds to the top of the graph at $C_0 = 1$. The dashed line is an average value for cells II to IV.

possible systematic errors. An equivalent fit to the theoretical data yielded $x_b^{th} = 0.232$ which is shown as a dotted horizontal line in the figure. We see that the uncertainties of x_b are largest for cells II and III, and that the data for those cells are consistent with the theoretical value. However, the more precise data for cells I and IV are clearly inconsistent with the theory. All four experimental results are consistent with $x_b \approx 0.185$ which is shown as a dashed horizontal line in the figure. For comparison, we mention that a similar fit to the data from cell J of Ref. [5] yielded $x_b = 0.14 \pm 0.54$ [26]. One sees that the uncertainty in these data is large enough for them to be consistent with the theory.

In order to learn more about the difference between theory and experiment, we considered the possibility that the theory might give the t dependence of R_b correctly, but that the amplitude is in error by a factor of order unity. Thus we fitted the function

$$R = C_0 \delta R_b^{th} + B_1 \tag{4}$$

to the data over the range $t \leq 10^{-4}$. Here B_1 represents the sum of the contributions from the experimental background R_B and from the unknown contribution $R_b^{th}(t_0)$ in the theory [see Eq. (2)]. Both C_0 and B_1 were least-squares adjusted. We found that the data could be fitted reasonably well, but with $C_0 < 1$. We obtained the values of C_0 shown in Fig. 3b (the error bars again are 95% confidence limits). The fit to the data is quite good, as shown by the dotted lines in Fig. 2. The data from cell J of Ref. [5] yielded $C_0 = 0.88 \pm 0.26$ from a similar fit [26]. As can be seen, the error bars based on these older data easily overlap the theoretical result $C_0 = 1$.

For all four cells, C_0 is significantly less than unity [12]. The results for cells II to IV are consistent with $C_0 \approx 0.535$, which is shown by the dashed line in Fig. 3b. This result is smaller than expected from estimates of the theoretical uncertainty of the amplitude of R_b^{th} . We do not know why the result for cell I is somewhat higher, but note that cell I was the only one which had a relatively rough and unpolished surface. To illustrate the quality of the fit to the data, we show $R_b = R - B_1$ and $C_0 \delta R_b^{th}$ for cells I and IV (which have the highest precision) in Fig. 4a, and the deviations of the data from $C_0 \delta R_b^{th}$ in Fig. 4b. We see that the fit to the data, although quite good, is not perfect, particularly for cell IV which yielded the most precise data.

We note that the theoretical result was obtained from a first-order perturbation calculation. It would be natural to look here for the difference between theory and experiment. However, the value of K in Eq. (2) is nearly independent of t , leaving the exponent of R_b essentially unaltered and nearly equal to that of ξ/λ . It seems unlikely that this situation would change at higher order. The value of K might change by a factor of order 1 and thus affect C_0 if a higher-order calculation were performed; but a change by a factor as small as 0.53 seems unlikely. This leaves us

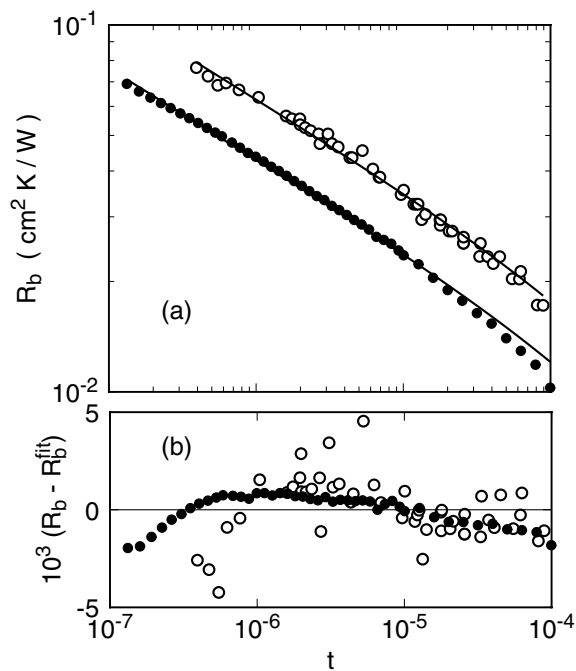


FIG. 4. (a) The contribution $R_b = R - B_1$ with B_1 determined from a fit of Eq. (4) to the data. Open (solid) circles are for cell I (IV) with $B_1 = 1.072(1.610)$. The solid lines are the fits. (b) Deviations of the data from the fits.

to suspect the choice of the DBC as the cause of the difference between the theory and the experiment. It would be very interesting to investigate theoretically whether the BC can be modified so as to cause the calculation to match the experimental results more closely.

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