Nusselt Number Measurements for Turbulent Rayleigh-Bénard Convection

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We present high-precision measurements of the Nusselt number $\mathcal{N}$ as a function of the Rayleigh number $R$ for a cylindrical sample of water (Prandtl number $\sigma = 4.4$) of height $L \approx 50$ cm and aspect ratio $\Gamma = D/L \approx 1$ ($D$ is the diameter) for $3 \times 10^9 \leq R \leq 6 \times 10^{10}$. For $R = 3 \times 10^9$ the data are consistent with existing results for acetone ($\sigma = 4.0$, $R \approx 3 \times 10^9$). There the measurements are also consistent with a model by Grossmann and Lohse (GL). As $R$ increases, the measurements fall below the GL prediction. Near $R = 6 \times 10^{10}$ the prediction is 8% above the data.

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Rayleigh-Bénard convection [1–3] (RBC) in a fluid heated from below is an important process in many respects. It occurs naturally in the Earth’s atmosphere and oceans and thus has short-term as well as long-term impact on climate and weather. It takes place in the Earth’s mantle and drives the motion of continental plates. It is the important heat transport mechanism in the outer layer of the Sun [4]. It plays a significant role in many industrial processes, where its enhancement or inhibition may have significant economic consequences. It is a common phenomenon in everyday life which provokes fascination for the nonspecialist. Last, but not least, it is one of the challenging and largely unsolved problems in nonlinear physics.

An important aspect of turbulent RBC is the global heat transport by the system [1–3]. This is usually expressed in terms of the Nusselt number

$$\mathcal{N} = QL/\lambda \Delta T,$$

(1)

where $Q$ is the heat-current density, $L$ the sample height, $\Delta T$ the applied temperature difference, and $\lambda$ the thermal conductivity in the absence of convection. Understanding the heat transport is important from a fundamental viewpoint because it is a central prediction of various theoretical models for this phenomenon [1–3,5–7]. Over time several models have been proposed to explain the dependence of $\mathcal{N}$ on the Rayleigh number

$$R = \alpha g TL^3/\kappa \nu$$

(2)

($\alpha$ is the isobaric thermal expansion coefficient, $\kappa$ the thermal diffusivity, and $\nu$ the kinematic viscosity) and the Prandtl number $\sigma = \nu/\kappa$ [1,5–7]. A model developed recently by Grossmann and Lohse [5], based on the decomposition of the kinetic and the thermal dissipation into boundary-layer and bulk contributions, provided an excellent fit to experimental data [8,9] for a cylindrical cell of aspect ratio $\Gamma \equiv D/L = 1$ when it was properly adapted [6] to the relatively small Reynolds numbers of the measurements. However, the data of Ref. [9] were used to determine five adjustable parameters of the model. Thus more stringent tests using data for the same $\Gamma$ but in parameter ranges other than those of Ref. [9] are desirable. A great success of the model was the excellent agreement with recent measurements by Xia et al. [10] for much larger Prandtl numbers than those of Ref. [9], at Rayleigh numbers near $1.78 \times 10^7$ and $1.78 \times 10^9$. Here we present new measurements for $\sigma = 4.4$ which extend to the heretofore unexplored range near $R = 6 \times 10^{10}$. As $R$ increases, we find a gradually increasing deviation of the data from the model prediction. For $R = 6 \times 10^{10}$ the prediction is already higher than the data by about 8%, and the trend of the data suggests that the difference would be larger at even larger $R$. It remains to be established experimentally whether the difference is due to a genuine breakdown of the theory, or whether it is attributable to a difference in boundary conditions for the measurements and the model [11].

One of the experimental problems in the measurement of $\mathcal{N}(R)$ is that the sidewall often carries a significant part of the heat current. Corrections for this effect are not easily made because of the thermal contact between the wall and the fluid and the consequent two-dimensional temperature field in the wall [12–15]. The present project was designed to provide data which extend the range of $R$ beyond that of Ref. [9] and which are not uncertain due to a significant sidewall correction. We used a classical fluid of relatively large conductivity confined by sidewalls of relatively low conductivity. The system of choice was water confined by Plexiglas with the greatest height $L$ and diameter $D$ permitted by other constraints. We built a convection cell with $L = 50.58$ cm with $D = 49.53$ cm, yielding an aspect ratio $\Gamma = 0.98$. The smallest $R$ for which accurate measurements could be achieved with this system was about $3 \times 10^9$, and reliable data up to $R = 6 \times 10^{10}$ were obtained. We estimated a wall correction [16] of only 0.3% for $R = 3 \times 10^9$ ($\mathcal{N} \approx 90$), and smaller corrections for larger $R$ (for comparison, at the same $R$ the same method of estimating the wall contribution [16] yields a correction of 11% for recent cryogenic helium data [14]). Based on this estimate we felt justified in neglecting the correction.
A schematic diagram of the apparatus is shown in Fig. 1. From bottom to top, we find first a catchpan (A) capable of containing up to 200 liters of fluid in case of a leak. Supported above it is a 81 cm diameter support plate (B) made of high-strength aluminum alloy. It stands on three legs [solid lines between (A) and (B)] which consist of threaded 1.27 cm diameter steel rods screwed into tapped holes in part (B). The entire apparatus could be leveled by adjusting these legs. Part (C) was a bottom adiabatic shield made of aluminum. It was supported above part (B) by a 50 cm diameter steel cylinder [vertical lines between (B) and (C)] with a height of 7.6 cm and a wall thickness of 0.32 cm. The central area of 49.5 cm diameter of the bottom of (C) was covered uniformly by parallel straight grooves of 0.76 cm depth and 0.40 cm width, interconnected by semicircles at their ends. Adjacent grooves were separated by 1.9 cm. Epoxied into the grooves was a heater made of AWG No. 15 Nichrome C resistance wire surrounded by fiberglas sleeving. This heater had a total length of 10 m and a resistance of 6.8 Ω. A second auxiliary heater with a 7 Ω resistance was wound around the outside of the shield. Suspended above the shield, on a steel cylinder of 3.8 cm height and 0.32 cm wall thickness, was the bottom plate (D) of the cell. It was made of high-strength aluminum alloy and had a thickness of 3.5 cm. Its top surface was finely machined, with tool marks of a depth less than 3 μm. It was coated by the “Tufram” process [17]. Its bottom surface contained the same type of Nichrome heater as the bottom shield. Its diameter was 54.6 cm. A top central section had a reduced diameter of 49.5 cm which was a close slide fit into the plexiglas sidewall cylinder (E). At one point of the side of the section of reduced diameter there was a vertical 0.16 cm diameter semicircular groove through which the fluid could enter the system. Five small holes were drilled from below at an angle into the bottom plate to within 0.32 cm of its top surface, and thermistors were mounted in these holes.

The plexiglas sidewall (E) had an inner diameter of 49.5 cm and a wall thickness of 0.63 cm. Its length was uniform around its circumference to ±0.01 cm and determined the aspect ratio of the cell. It extended 2.38 cm below the top surface of the bottom plate. An ethylene-propylene O-ring sealed the system from the outside well below the top surface of the bottom plate. A similar construction was used to terminate the cell wall at the top. This construction minimizes the heat flow into the wall and provides a well-defined geometry for simulations of this heat-flow problem.

The sidewall was surrounded by an adiabatic side shield (F) made of aluminum. Epoxied to the outside of this shield was a double spiral consisting of 15 m of aluminum tubing. Water from a temperature controlled circulator flowed through the tubing. The shield was suspended above the support plane (B) by six 1.9 cm diameter and 17.5 cm long plastic rods (not shown in Fig. 1). During measurements its temperature was kept at the mean temperature of the system.

The top of the cell was provided by an aluminum top plate (H) which was similar to the bottom plate in its dimensions, was also Tufram plated, but did not contain a heater. It contained an outlet for the fluid which was identical to the inlet in the bottom plate, but in the cell assembly care was taken to locate the outlet at an angular position opposite to the inlet. The top plate was cooled by temperature controlled water from a refrigerated circulator. It had a thickness of 3.34 cm, and a double-spiral water-cooling channel was machined directly into it. The channel width and depth were 0.95 × 2.54 cm². The spacing between adjacent turns of the spiral was 2.54 cm. The bottom of the groove came within 0.79 cm of the aluminum-fluid interface. An additional plate was “O”-ring sealed to the top plate from above to close the spiral channel. Small holes were drilled through the two-plate composite from above to within 0.32 cm of the aluminum-fluid interface, and calibrated thermistors were mounted with their heads within 0.48 cm of the convecting fluid. Each thermistor was protected from circulating water by an additional small O-ring between the two plates. To avoid convection of air in the vicinity of the cell, the entire space outside the cell but inside the dashed rectangle K was filled with low-density (firmness rating 1) polyurethane foam sheet.

The system contained 16 thermistors [18] that were calibrated simultaneously in a separate apparatus against a laboratory standard based on a standard platinum thermometer. Deviations of the data from the fit were generally less than 0.002 °C. In the top as well as the bottom plate one thermistor terminated at the plate center and four were located equally spaced on a circle of 43 cm diameter. The remaining thermistors were mounted on the adiabatic bottom shield, the adiabatic side shield, and
the outside of the plexiglas cell wall. The power of the bottom-plate heater was determined by a four-lead method. The cell was filled with deionized degassed water. The cell was leveled to better than 0.1°.

The system was equilibrated with both top and bottom at 40 °C for over a day. Thereafter the power needed to maintain the bottom-plate temperature was constant and equal to 0.1 W. We do not know the origin of this parasitic heat loss, but subtracted it from all subsequent measurements. It was 0.4% (0.02%) of $Q$ for $\Delta T = 1(10)$ °C. A correction to the temperature difference $\Delta T = T_b - T_t$ between the bottom and top plates was made for the temperature change across the aluminum layers between the fluid and the thermistor heads. The correction was estimated to be 0.8% (1.5%) of $\Delta T$ for $\Delta T = 1(10)$ °C. The measurements were made at mean temperatures close to 40.00 °C where the Prandtl number $\sigma$ was 4.38. The variation of $\sigma$ over the applied temperature difference can be estimated from $(1/\sigma)(d\sigma/dT) = 0.020$.

An important issue is the extent to which the system can be described by the Oberbeck-Boussinesq approximation (OBA) [19,20]. Several parameters have been used to describe non-Oberbeck-Boussinesq (NOB) effects, but it is not known how large these parameters may be in order for the Nusselt number to represent a Boussinesq system within a given error margin. The parameter $\eta$ was defined by Busse [21] to describe NOB effects near the onset of convection [22]. It may not be too relevant to turbulent convection; but we list it nonetheless in Table I. Another parameter is the relative change in density of the fluid which is given by $\alpha = \frac{1}{\sigma}$. It is listed in the fourth column of Table I. Finally, and perhaps most relevant to the problem at hand, there is an estimate of the ratio of the temperature drop across the top boundary layer to that across the bottom one [24]. This ratio $x_{WL}$ is also given in Table I [25]. The largest $\Delta T$ used in the measurements was about 13 °C.

In Fig. 2 we show our results as solid circles on a double logarithmic scale. Also shown, as open circles, are the data for acetone from Ref. [8]. Their range of $R$ only barely overlaps with that of the present data, but there is consistency. The solid line is the prediction of the GL model. As discussed above, this model was fitted to data which included the acetone results. It fits the acetone data reasonably well. Our new data depart from the prediction more and more as $R$ increases. For $R = 6 \times 10^{10}$ the difference is 8%.

The quantity $\mathcal{N}/R^{0.3}$ is nearly constant. Thus the relationship between the data and the prediction can be shown with greater resolution when the results are presented in a “compensated” plot of $\mathcal{N}/R^{0.3}$ vs $R$. This is done in Fig. 3.

![FIG. 3](image.png)

TABLE I. The Rayleigh number $R$, and several measures of departures from the Boussinesq approximation (see text), at two temperature differences $\Delta T$. The mean temperature is 40 °C.

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In the study of the system of Ref. [6], we found the effective exponent $\gamma_{eff} = d[\log(\mathcal{N})]/d[\log(R)]$ decreases very slightly with increasing $R$ over the range

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The quantity $\mathcal{N}/R^{0.3}$ is nearly constant. Thus the relationship between the data and the prediction can be shown with greater resolution when the results are presented in a “compensated” plot of $\mathcal{N}/R^{0.3}$ vs $R$. This is done in Fig. 3. We see also on this scale that the acetone data are consistent with the new measurements using water (solid circles). Here it is interesting to note that the acetone data without the wall correction (plusses in Fig. 3) are inconsistent with the water data which do not require a wall correction. The new data depart systematically from the prediction as $R$ increases. If the data are fitted locally to an effective power law $\mathcal{N} = N_0 R^{\gamma_{eff}}$, then we find reasonable agreement between the GL model and the wall-corrected acetone data near $R = 10^9$ with $\gamma_{eff} = 0.295$ for the data as well as the model. Near $R = 2 \times 10^{10}$, on the other hand, we get $\gamma_{eff} = 0.310$ for the model and $\gamma_{eff} = 0.291$ for the measurements.

In this Letter, we presented new high-precision measurements of the Nusselt number $\mathcal{N}$ as a function of the Rayleigh number $R$ using water with a Prandtl number $\sigma = 4.38$ over the range $3 \times 10^9 < R < 6 \times 10^{10}$. Our results are consistent with previous measurements at nearly the same $\sigma$ using acetone [8], but at large $R$ they disagree with the prediction of the model of Grossmann and Lohse. The acetone and water data suggest that the effective exponent $\gamma_{eff} = d[\log(\mathcal{N})]/d[\log(R)]$ decreases very slightly with increasing $R$ over the range

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