Heat transport and the large-scale circulation in rotating turbulent Rayleigh–Bénard convection

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Measurements of the Nusselt number $Nu$ and of properties of the large-scale circulation (LSC) for turbulent Rayleigh–Bénard convection are presented in the presence of rotation about a vertical axis at angular speeds $0 \leq \Omega \lesssim 2 \text{rad s}^{-1}$. The sample chamber was cylindrical with a height equal to the diameter, and the fluid contained in it was water. The LSC was studied by measuring sidewall temperatures as a function of azimuthal position. The measurements covered the Rayleigh-number range $3 \times 10^8 \lesssim Ra \lesssim 2 \times 10^{10}$, the Prandtl-number range $3 \lesssim Pr \lesssim 6.4$ and the Rossby-number range $0 \leq (1/Ro \propto \Omega) \lesssim 20$. At modest $1/Ro$, we found an enhancement of $Nu$ due to Ekman-vortex pumping by as much as 20%. As $1/Ro$ increased from zero, this enhancement set in discontinuously at and grew above $1/Ro_c$. The value of $1/Ro_c$ varied from about 0.48 at $Pr = 3$ to about 0.35 at $Pr = 6.2$. At sufficiently large $1/Ro$ (large rotation rates), $Nu$ decreased again, due to the Taylor–Proudman (TP) effect, and reached values well below its value without rotation. The maximum enhancement increased with increasing $Pr$ and decreasing $Ra$ and, we believe, was determined by a competition between the Ekman enhancement and the TP depression. The temperature signature along the sidewall of the LSC was detectable by our method up to $1/Ro \approx 1$. The frequency of cessations $\alpha$ of the LSC grew dramatically with increasing $1/Ro$, from about $10^{-5} \text{s}^{-1}$ at $1/Ro = 0$ to about $2 \times 10^{-4} \text{s}^{-1}$ at $1/Ro = 0.25$. A discontinuous further increase of $\alpha$, by about a factor of 2.5, occurred at $1/Ro_c$. With increasing $1/Ro$, the time-averaged and azimuthally averaged vertical thermal gradient along the sidewall first decreased and then increased again, with a minimum somewhat below $1/Ro_c$. The Reynolds number of the LSC, determined from oscillations of the time correlation functions of the sidewall temperatures, was constant within our resolution for $1/Ro < 0.3$ and then decreased with increasing $1/Ro$. The retrograde rotation rate of the LSC circulation plane exhibited complex behaviour as a function of $1/Ro$ even at small rotation rates corresponding to $1/Ro < 1/Ro_c$.

Key words: Bénard convection, rotating turbulence, turbulent convection

1. Introduction

Understanding turbulent convection in a fluid confined between two horizontal plates and heated from below (Rayleigh–Bénard convection (RBC); for reviews, see e.g. Kadanoff 2001; Ahlers, Grossmann & Lohse 2002, 2009; Ahlers 2009; Lohse & Xia 2010) remains one of the challenging problems in nonlinear physics.

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It is well established that a major component of the dynamics of this system is a large-scale circulation (LSC). For a cylindrical system of height $L$, which is almost equal to diameter $D$ (aspect ratio $\Gamma \equiv L/D \approx 1$), the LSC takes the form of a single convection roll. It has some particularly interesting properties. The dynamics of its near-vertical circulation plane can be described well as a stochastically driven system, with the driving due to the smaller scales of the turbulent flow (Brown & Ahlers 2007a, 2008b). Most of the time, the orientation of the circulation plane undergoes azimuthal diffusion (Sun, Xi & Xia 2005; Xi, Zhou & Xia 2006; Brown & Ahlers 2006a,b). In addition, it has a number of interesting features, including torsional oscillations (Funfschilling & Ahlers 2004; Funfschilling, Brown & Ahlers 2008), a ‘sloshing’ mode (Brown & Ahlers 2009; Xi et al. 2009; Zhou et al. 2009) and sudden rotations of the azimuthal orientation (Cioni, Ciliberto & Sommeria 1997; Brown & Ahlers 2006b). Occasionally, there are cessations of the circulation (where the flow stops completely), followed by a re-birth of the circulation at a random new orientation (Brown, Nikolaenko & Ahlers 2005a; Brown & Ahlers 2006b). All of these features are explained well by the stochastic model of Brown & Ahlers (2007a, 2008b).

A second important aspect is the existence of thin thermal and viscous boundary layers (BLs) just above the bottom and just below the top plate. Even when the interior of the system is highly turbulent, these BLs, although fluctuating in time, are laminar (Zhou & Xia 2010) and can be described well by the classical Prandtl–Blasius boundary-layer theory. The thermal BLs control the global heat transport of the system (Ciliberto, Cioni & Laroche 1996; Ahlers, Brown & Nikolaenko 2006b). The heat transport is usually expressed in terms of the ratio of the effective thermal conductivity of the convecting fluid to the diffusive thermal conductivity $\lambda$ of the quiescent fluid known as the Nusselt number:

$$Nu = \frac{QL}{A\Delta T\lambda}.$$  \hspace{1cm} (1.1)

Here $Q$ is the heat current carried by the fluid, $A$ is the cross-sectional area of the sample, $L$ is the sample height and $\Delta T$ is the applied temperature difference. For a reasonable approximation, about half of $\Delta T$ is found across each thermal BL, and the sample interior, on average, is nearly isothermal (see e.g. Brown & Ahlers (2007b) and §4). Over the parameter range of interest for the present work, the dependence of $Nu$ on the Rayleigh number

$$Ra \equiv \frac{\alpha g \Delta T L^3}{\kappa \nu}$$  \hspace{1cm} (1.2)

($\alpha$ is the isobaric thermal expansion coefficient, $g$ is the acceleration of gravity, $\kappa$ is the thermal diffusivity and $\nu$ is the kinematic viscosity) and on the Prandtl number

$$Pr \equiv \nu/\kappa$$  \hspace{1cm} (1.3)

is explained rather well by the model of Grossmann and Lohse (GL) (Grossmann & Lohse 2000, 2001).

In the present paper, we address the question of how the LSC and the heat transport are modified when the sample is rotated about a vertical axis. The influence of the Coriolis force due to rotation is of importance in numerous astrophysical and geophysical phenomena, including convection in the Arctic and Antarctic oceans (see e.g. Marshall & Schott 1999; Gascard et al. 2002), Earth’s outer core (see e.g. Glatzmaier et al. 1999; Jones 2000), the interior of gaseous giant planets (see e.g. Busse 1994) and the outer layer of the Sun (see e.g. Miesch 2000).

In the present paper, we focus on a parameter range well above onset, where the interior of the RBC system is turbulent. Much prior work exists as well in this regime. In a previous paper, Brown & Ahlers (2006a) discussed the very weak influence of Earth’s rotation on the LSC in turbulent RBC. Here we study the effect of deliberate rotation about a vertical axis. It is worth noting that any rotation frequency $f$ imposed in the laboratory (which typically will be larger than $10^{-3}$ Hz or so) will be two or more orders of magnitude larger than the Earth’s rotation rate of about $10^{-5}$ Hz. The effect of deliberate rotation has been investigated before by numerous experiments, including those of Rossby (1969), Boubnov & Golitsyn (1986), Boubnov & Golitsyn (1990), Fernando, Chen & Boyer (1991), Zhong et al. (1993), Liu & Ecke (1997b), Sakai (1997), Vorobiev & Ecke (1998), Hart & Olsen (1999), Hart, Kittelman & Ohlsen (2002), Vorobiev & Ecke (2002), Kunnen, Clercx & Geurts (2008a, b), Zhong et al. (2009), Stevens et al. (2009), Liu & Ecke (2009), King et al. (2009), Kunnen, Geurts & Clercx (2010) and Niemela, Babuin & Sreenivasan (2010). It has also been studied, by using direct numerical simulations (DNS), by Julin et al. (1996a, b), Kunnen, Clercx & Geurts (2006, 2008b), Zhong et al. (2009), Stevens et al. (2009), Schmitz & Tilgner (2009), King et al. (2009), Kunnen et al. (2010), Mishra et al. (2010) and Stevens, Clercx & Lohse (2010a, b). For a summary of much of this work, we refer the reader, for instance, to the more detailed introduction of the recent paper by Kunnen et al. (2010). The relationship of our work to some of this sizable literature will be discussed in §6.

Here we report on an experimental study of a cylindrical sample with aspect ratio $\Gamma \approx 1$ as a function of the rotation rate $\Omega = 2\pi f$ ($f$ is the rotation frequency in Hz) over the ranges $3 \times 10^8 \lesssim Ra \lesssim 2 \times 10^{10}$ and $3.0 \lesssim Pr \lesssim 6.4$. The dimensionless rotation rates are expressed in various forms in the literature, including the Ekman number

$$Ek = \frac{\nu}{L^2} \frac{1}{\Omega},$$

the convective Rossby number (henceforth simply referred to as the Rossby number, but distinct from the inertial Rossby number, which will not appear in this paper;
Rotating turbulent Rayleigh–Bénard convection

<table>
<thead>
<tr>
<th>Run</th>
<th>$T_m$ (°C)</th>
<th>$\Delta T$ (K)</th>
<th>$Pr$</th>
<th>$10^{-9} Ra$</th>
<th>$Ro \ \Omega$</th>
<th>$10^5 Ek \ \Omega$</th>
<th>$10^{-10} Ta/\Omega^2$</th>
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<tr>
<td>E1</td>
<td>23.00</td>
<td>4.68</td>
<td>6.41</td>
<td>1.21</td>
<td>0.1048</td>
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<td>1.716</td>
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<tr>
<td>E2</td>
<td>24.00</td>
<td>0.996</td>
<td>6.26</td>
<td>0.273</td>
<td>0.0493</td>
<td>1.494</td>
<td>1.793</td>
</tr>
<tr>
<td>E3</td>
<td>24.00</td>
<td>7.96</td>
<td>6.26</td>
<td>2.19</td>
<td>0.1396</td>
<td>1.494</td>
<td>1.793</td>
</tr>
<tr>
<td>E4</td>
<td>40.00</td>
<td>0.996</td>
<td>4.38</td>
<td>0.563</td>
<td>0.0618</td>
<td>1.091</td>
<td>3.364</td>
</tr>
<tr>
<td>E5</td>
<td>40.00</td>
<td>1.99</td>
<td>4.38</td>
<td>1.13</td>
<td>0.0875</td>
<td>1.090</td>
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<tr>
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<td>0.790</td>
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<td>7.99</td>
<td>0.2021</td>
<td>0.790</td>
<td>6.414</td>
</tr>
</tbody>
</table>

Table 1. Parameters for all experimental runs. The last three columns give the products ($Ro \ \Omega$) (rad s$^{-1}$), ($Ek \ \Omega$) (rad s$^{-1}$) and ($Ta \ \Omega^{-2}$) (s$^2$ rad$^{-2}$), which are constant material parameters for a given experimental run.

Most of our measurements were done at several constant $Pr$ and $Ra$, varying $\Omega$ (see table 1). We find that the observed phenomena are best described by using $Ro$, but prefer to use its inverse $1/Ro$ in order to have a parameter proportional to the rotation rate. Our measurements extend from $1/Ro = 0$ (no rotation) to $1/Ro \simeq 20$. More information about various parameter ranges is reported in table 1. We report results for the Nusselt number, as well as for the effect of rotation on properties of the LSC. Some of our results have been reported briefly before (Stevens et al. 2009; Zhong et al. 2009) in two publications that described the results of a very fruitful collaboration with the groups of H. J. H. Clercx and D. Lohse, who carried out DNS of this system over parameter ranges that overlapped with ours and revealed excellent agreement with the experiment.

We (as well as others) found that modest rotation enhances the heat transport in this system due to Ekman-vortex pumping, but that strong rotation decreases the Nusselt number. The decrease is usually attributed to the Taylor–Proudman ‘theorem’ (Tritton 1988), which states that velocity gradients parallel to the rotation axis, and thus also the convected heat transport, are suppressed. We remark that strictly this ‘theorem’ is valid only when nonlinear, dissipative and buoyancy terms are absent in the equation of motion. When the Coriolis force is much larger than the inertial and buoyancy forces as represented by the nonlinear and buoyancy terms in the Boussinesq equation, then $1/Ro$ (as well as the inverse inertial Rossby number) is much greater than one. When the dissipative term is also much less than the Coriolis term, then $1/Ek \gg 1$. When all of those conditions pertain, then the TP ‘theorem’ becomes a good approximation and the phenomenon dominates; but since the ‘theorem’ is no longer exact, we prefer to refer to the phenomenon as the TP ‘effect’. This effect is also present at modest rotation, but there it does not dominate. Our Nusselt-number
measurements, shown below in figures 4, 8 and 20, indicate for instance that for $Pr \simeq 4.4$ the TP effect can become noticeable when $1/Ro \simeq 2$; for $Pr \simeq 6.3$ it shows up measurably only when $1/Ro \simeq 7$. When the strength of the TP effect increases to the point where it dominates completely, then $Nu$ is suppressed completely and the onset of convection is reached from above. As we shall see below in figure 9, none of the measurements in the current paper comes close to this latter condition.

In the next section, we describe the apparatus, experimental methods, and procedures in some detail. Section 2.1 discusses the construction of the rotating table and the sample that is mounted on it. The thermometry involved in our work is explained in §2.2. In §2.3, we give details of the general experimental protocol. In §2.4, we explain how high-precision Nusselt-number measurements were made. In §2.5, we describe the sidewall temperature measurements that were used to obtain information about the LSC. Results for $Nu$ are presented in §3, first for $\Omega = 0$ and then for $\Omega > 0$. Measurements of the temperature gradient along the sidewall are presented in §4. Several properties of the LSC are described in §5. These include, in sequence, the associated Reynolds number $Re$, the time-averaged temperature amplitude $\langle \delta \rangle$, the size of the root-mean-square temperature fluctuations $\langle \delta T \rangle$ about a smooth azimuthal cosine variation, the occurrence of cessations and the dependence of the cessation frequency $\alpha$ on $1/Ro$, and various aspects of the retrograde rotation rate of the LSC circulation plane. A summary and discussion of the results is provided in §6.

2. Experimental apparatus and methods

2.1. The rotating table and the convection system

A schematic diagram and a photograph of the apparatus are shown in figure 1. A heavy aluminum plate A rested securely on the laboratory floor. A second plate of size 50 cm $\times$ 50 cm was the base plate of the static support structure B of the rotating table (in the schematic only the plate is shown) and was supported on three equally spaced screws C (only one is shown). The screws were used to level the support structure B to within $10^{-4}$ rad. The rotation axis D of the table was accurately orthogonal to B and carried the rotating plate E. It was driven by a superior electric model M093-FC11 stepping motor operated in a microstepping mode. All other mechanical
details of the table construction are omitted here as they are not essential to the table performance. Supported by three further screws G was the base plate F of the convection system. This system was nearly the same as that described elsewhere (Brown et al. 2005b), and here we shall be brief. The screws G were used to ensure that the upper surface of the sample bottom plate L was level within $10^{-4}$ rad. Supported on F by a stainless-steel ring was a thermal shield I. The bottom of I was covered by a heater, and a thermistor was located at the centre of the upper surface of I. A second thermistor and heater were located near the periphery of I; unfortunately, insufficient electrical leads were available in the rotating frame to allow this extra thermal protection to be used during most of this project. This compromise did lead to a small deterioration in the accuracy of the Nusselt-number measurements, but did not affect the precision and thus had no influence on the ratios of $Nu$ with and without rotation. In a later version of the apparatus, more electrical feed-throughs were installed and this auxiliary bottom-shield heater was put to use once more.

The copper bottom plate L was supported on a second stainless-steel ring K and had an anvil on its upper side that protruded into the Plexiglas sidewall M. Plate L had a heater covering its bottom surface as described before. The copper top plate O rested on and also protruded into the sidewall, and had in it a double-spiral water-cooling channel. The water circuit was brought into the rotating frame through the feed-through (P, Q). The rotor P of the feed-through was supported by a structure R from the rotating base plate F, and the stator Q was supported similarly via S from the stator plate B. A dual Dynamic Sealing Technologies model GP-321-E-ESM428 water feed-through (sufficient for a single water circuit) worked in conjunction with feed-throughs for electrical leads. Additional electrical feed-throughs were available at the bottom of the system through the rotation axis D. In a later version of the apparatus, the water feed-through was replaced by a quadruple Scott Rotary Seals model AP-341 four-passage feed-through, combined with an ES-12 slip ring for electrical leads. This unit could accommodate two water circuits. Both types of water feed-throughs were supposed to be suitable for use with water as the fluid, but after extended use began to leak because of corrosion and associated abrasion. Thus, in a later version of the apparatus, we used the Scott Rotary Seals unit with ethylene glycol (EG) and an anti-corrosion additive. The total height of the apparatus was approximately 2 m.

Thermal protection towards the side was provided by a concentric thermal shield N. In the original design, this shield was cooled or heated by a separate water circuit. In the rotating frame, we only had a single water circuit available, and it was needed for the top plate. Thus the shield N was temperature controlled at its mid-height by an electrical heater that was wound uniformly along its length. This arrangement precluded the operation of the system below the ambient laboratory temperature, which was near 22°C. The temperature uniformity of the shield was not as good as it had been with the water cooling, and this contributed further, albeit only slightly, to a deterioration of the Nusselt-number accuracy. In a later version of the apparatus, the side shield was converted back to water or EG cooling/heating.

Most of the volume inside R that was not occupied by water or structural members was filled with low-density open-pore foam as shown in the photograph. The sample had two fill lines, one entering between the bottom plate anvil and the sidewall and the other entering similarly at the top plate. After complete filling of the sample, these fill lines were terminated in two small plastic containers, as shown in the photograph near the arrow from R. The pressure in these containers was referenced to the atmosphere by virtue of a small hole in the top of each.
2.2. Thermometry and heaters

The five thermometers in the bottom and the five in the top plate, as well as one thermometer on the bottom and one on the side shield, were calibrated in a separate calibration facility with a precision of a millikelvin against a Hart Scientific Model 5626 platinum resistance thermometer, which in turn had been calibrated against various fixed points on the ITS-90 temperature scale by the Hart Scientific Division of Fluke Corporation. Consistency between all of these thermometers is excellent; at the same temperature, their readings all agree within ±0.002 K.

The 20 sidewall thermometers (see §2.5) were calibrated against the top- and bottom-plate thermometers after they had been installed in the sidewall. The plate temperatures were set so that \( \Delta T = 0.1 \) K to ensure equilibration of the system in a reasonable time (half a day or so), and the sidewall thermometers were then assumed to be at the mean temperature \( T_m \) of the top and bottom plates.

The main bottom-plate heater was driven by an Agilent Model 6675 DC power supply. The current was taken to be given correctly by the power supply, and the voltage across the heater was measured for each data point with a second pair of leads. The bottom-shield and side-shield heaters were driven by Agilent Model E3634 DC power supplies using a two-lead method. All three heaters were operated in a digital feedback loop in conjunction with the relevant thermometer so as to hold the temperature in question constant.

2.3. Typical experimental protocol

Measurements using water as the convecting fluid and at several mean temperatures ranging from 23 \((Pr = 6.41)\) to 60°C \((Pr = 3.05)\) were made for a cylindrical sample with \( \Gamma \approx 1 \). The sample had a height \( L = 24.78 \pm 0.05 \) cm and a diameter \( D = 24.78 \pm 0.05 \) cm, yielding an aspect ratio \( \Gamma = D/L = 1 \). The rotation rates were kept below \( f = 0.3 \) Hz, yielding Froude numbers \( Fr = \Omega^2(D/2)/g \) less than 0.05 for all runs and much smaller for most.

Readings of all thermometer resistances (see §2.2) and of the bottom-plate heater current and heater-voltage were taken about every 3 s. The top (bottom) temperature \( T_t (T_b) \) was set equal to the average of the five thermometers embedded in the top (bottom) plate. For any given data point, measurements over typically the first 6 hours were discarded to avoid transients, and data taken over an additional period of at least another 6 hours were averaged to get the heat-current density \( Q, T_b \) and \( T_t \). A near-negligible correction of \( T_t \) and \( T_b \) for the small temperature drops in the plates between the thermistors and the copper-fluid interfaces was made. No corrections were made for the finite conductance of the sidewall, as this had been shown to be negligible for the present conditions (Nikolaenko et al. 2005).

In nearly all cases, a sequence of measurements was carried out at constant \( \Delta T \), varying the angular speed \( \Omega \). Parameters for all runs are listed in table 1. Rayleigh numbers were determined from the temperature differences \( \Delta T = T_b - T_t \), and from the fluid properties evaluated at \( T_m = (T_t + T_b)/2 \). For each \( Pr \) and \( Ra \) studied with rotation the Nusselt number was also determined without rotation, i.e. for \( \Omega = 0 \), and then the corresponding ratios \( Nu(\Omega)/Nu(0) \) were computed. In order to eliminate the influence of the very small variations of \( Ra \) between different points in a given run at nominally constant \( Ra \), we first computed \( Nu(\Omega)/Ra^{0.3} \) and \( Nu(0)/Ra^{0.3} \), which are nearly independent of \( Ra \), and then used their ratio as the best estimate of \( Nu(\Omega)/Nu(0) \).

In some cases, much longer runs were made in order to obtain better statistics on the properties of the LSC from the sidewall thermometers.
2.4. Nusselt-number measurements

Because of the imperfect temperature uniformity of the bottom shield (I in figure 1) discussed in §2.1, there was a small parasitic heat loss $Q_p$ from the bottom plate that depended on $T_b$. This had little influence on $Nu$ at large $Ra$, where $Q$ was large, but caused systematic errors at the smaller values of $Ra$. We determined $Q_p(T_b)$ by running the experiment with various $T_m$ and with the very small $\Delta T = 0.1$ K. Using $\Delta T = 0.1$ K for this purpose was preferable because for $\Delta T = 0$ the thermal relaxation times of the system were too long to reach a stationary state in a reasonable time. With $\Delta T = 0.1$ K, nearly 10 h were required to reach a stationary state. Each run was conducted for one day and the data after about 16 h were used to evaluate $Q$, $T_b$ and $T_t$. Under those conditions, we had $4 \times 10^7 \lesssim Ra \lesssim 9 \times 10^7$ depending on $T_m$, and expected $Nu/Ra^{0.304} \approx 0.123$ ($Nu \approx 24$). We adjusted $Q_p$ until a corrected current $Q = Q_{appl} - Q_p$ ($Q_{appl}$ is the Joule heating applied to the bottom plate) gave the expected results for $Nu$ at this very small $\Delta T$, where the result was very sensitive to $Q_p$. In figure 2, we show $Q_p$ as a function of $T_b$. A quadratic polynomial fitted the data well, and was used to make the correction. We note that $Q_p$ vanished just about at the ambient laboratory temperature, as it should. At other temperatures, it ranged up to about 0.8 W. Measurements of $Nu$ used in the remainder of this paper were made with $1 \lesssim \Delta T \lesssim 32$ K, where $7 \lesssim Q \lesssim 600$ W; one sees that the correction for $Q_p$ was small enough to be applied with confidence or even negligible.

A small correction to $Nu$ was applied for the finite conductivity of the copper top and bottom plates. The size of this correction had been measured previously (Brown et al. 2005b); applying it increased $Nu$ by 1.3% at the largest $\Delta T = 32$°C ($Ra = 1.8 \times 10^{10}$), by 0.3% at $\Delta T = 4$°C ($Ra = 2.2 \times 10^9$), and had little influence at lesser values of $\Delta T$. It is unknown whether and how rotation may influence this correction.

2.5. Sidewall-temperature measurements

As discussed in previous publications (Brown et al. 2005a; Brown & Ahlers 2006a,b; Brown, Funfschilling & Ahlers 2007; Funfschilling et al. 2008), three sets of thermistors were inserted into blind holes in the sidewall so as to be within a millimetre or so of the fluid without penetrating into it. The three sets were at vertical positions $z = -L/4$, 0 and $L/4$ (we take the origin of the up-pointing vertical $z$-axis at the horizontal mid-plane of the sample). We shall refer to the levels as $b$ for bottom at $z = -L/4$, $m$ for middle at $z = 0$ and $t$ for top at $z = L/4$. At each level, there were eight such thermistors equally spaced azimuthally. For the bottom set we were able to use only four of these because our total number of electrical feed-throughs out of the rotating frame was limited (after completion of the work discussed in the present paper this deficiency was remedied). The thermistors were able to sense the...
3. Results for the Nusselt number

All measurements of $Nu$ ($Ra$, $Pr$, $Ro$) may be found in numerical form in a separate document submitted with this paper as supplementary material.

3.1. Nusselt numbers for $\Omega = 0$

In figure 3(a), we show the results for $\Omega = 0$ in the reduced form $Nu/Ra^{0.304}$ as a function of $Ra$ on a logarithmic scale. Also shown are the 'strictly Boussinesq' data adjacent fluid temperature without interfering with delicate fluid-flow structures. We measured the temperature of each thermistor with a sampling period of about 3.5 s. Since the LSC carried warm (cold) fluid from the bottom (top) plate up (down) the sidewall, these thermistors detected the location of the upflow (downflow) of the LSC by indicating a relatively high (low) temperature.

To determine the orientation and strength of the LSC, we fit the function

$$T_f = T_{w,m} + \delta_m \cos \left( \frac{i\pi}{4} - \theta_m \right), \quad i = 0, \ldots, 7,$$

separately at each time step, to the eight temperature readings $T_i$ obtained from the thermistors at height $z = 0$, least squares adjusting $T_{w,m}$, $\delta_m$ and $\theta_m$. The fit parameter $\delta_m$ is a measure of the temperature amplitude of the LSC and $\theta_m$ is the azimuthal orientation of the plane of the LSC circulation. As defined here, the orientation $\theta_m$ is on the side of the sample, where the LSC is warm and up-flowing and is measured relative to the location of thermometer zero. We calculated mean temperature $T_{w,t}$, orientation $\theta_t$, and amplitude $\delta_t$ for the top level at $z = L/4$ separately by the same method as for the middle row. For the bottom row (where only four equally spaced thermometers could be read), we calculated their average temperature $T_{w,b}$.
from Funfschilling et al. (2005) for $Pr = 4.38$. One can see that most of the present data for $Pr = 4.38$ (open circles) fall about half a per cent below the reference data. This is within the possible systematic errors of the two data sets, due for instance to uncertainties in the sample heights and diameters (see Funfschilling et al. 2005). At the largest $Ra$ there is a negative contribution from non-Boussinesq effects (Ahlers et al. 2006a), which at the largest $Ra$ becomes as large as 0.7%. No correction was made for this effect. Also shown, as a dotted line (red online), is the model of Grossmann & Lohse (2001) for $Pr = 4.38$. The overall agreement with the data is quite good, although the $Ra$-dependence seems slightly different.

Note that the apparent ‘scatter’ of the points, most noticeable near $Ra = 1 \times 10^9$, is actually not scatter at all, but rather reveals a small dependence of $Nu$ on $Pr$. In order to illustrate this in more detail, we show some of the data from figure 3(a) in figure 3(b) as a function of $Pr$. There we divided the results by the experimental measurements at $Pr = 4.38$. One sees that, for $Ra = 1.2 \times 10^9$ (solid circles) and for $Ra = 2.2 \times 10^9$ (open circles), $Nu$ decreased very slightly as $Pr$ increased. This can be described by an effective power-law-dependence $Nu \propto Pr^\alpha$ with $\alpha \approx -0.01$, as shown by the solid line in the figure. This is a weaker dependence on $Pr$ than was reported earlier (Nikolaenko et al. 2005), and shown by the open squares and the dashed line, for a sample with $\Gamma = 0.66$ and $Ra \approx 10^{11}$. It is, however, quite consistent with earlier measurements for $\Gamma = 1.00$ (Ahlers & Xu 2001; Xia, Lam & Zhou 2002) over a much wider range of $Pr$. The new data are also consistent with those of Roche et al. (2002), which according to those authors imply at most a very mild dependence of $Nu$ on $Pr$, corresponding to $|\alpha| \leq 0.03$. Our new data imply that the expected maximum in $Nu(Pr)$ (Grossmann & Lohse 2001) occurs at or below $Pr = 3$, whereas the GL model gives a maximum near $Pr = 5$ as shown by the dotted line (red online) in figure 3(b). Most likely the small differences between the model and the data could be eliminated by an adjustment of the model parameters.

We expect that any small systematic errors in $Nu$ will be essentially the same with and without rotation, and thus that the ratio $Nu(\Omega > 0)/Nu(\Omega = 0)$ will not be influenced by them. The precision of the data for $Nu$ depends somewhat on the size of $\Delta T$ and on the duration of the run over which measurements are averaged. For $\Delta T \gtrsim 4$ K it is believed to be near 0.1%. For smaller $\Delta T$ somewhat larger systematic errors, independent of $\Omega$, are possible because of systematic uncertainties in the temperature scales of the top- and bottom-plate thermometers. However, these errors should not affect the ratio $Nu(\Omega)/Nu(0)$. For the smallest $\Delta T \approx 1$ K random errors due to temperature resolution are expected to be a few tenths of 1%.

### 3.2. Nusselt numbers for $\Omega > 0$

#### 3.2.1. Qualitative features

Two representative sets of data for $Nu(\Omega)/Nu(0)$ as a function of $1/Ro$ on a logarithmic scale are shown in figure 4. One sees that there are several distinct ranges, with different characteristic variations of $Nu(\Omega)$ with $\Omega$ or $1/Ro$. To the left of the short vertical dotted line there was very little change of $Nu$ with $1/Ro$, although as we shall see in §3.2.2, $Nu$ was not strictly constant. At the location of the vertical dotted line, which we shall call $1/Ro_c$, there was a sharp transition, or bifurcation, to a different state where $Nu$ initially increased dramatically and then decreased strongly with increasing $1/Ro$. At the bifurcation, $Nu(\Omega)$ was continuous; thus the bifurcation was supercritical. This phenomenon has been reported before by Stevens et al. (2009), and upon close inspection it can already be seen in the DNS of Kunnen, Clercx & Geurts (2008b). As $1/Ro$ increased beyond $1/Ro_c$, $Nu$ first increased linearly with
Figure 4. The ratio $\frac{Nu(\Omega)}{Nu(\Omega = 0)}$ as functions of $1/Ro$ on a logarithmic scale. Solid circles, run E6, $Ra = 2.25 \times 10^9$ and $Pr = 4.38$. Open circles, run E2, $Ra = 2.73 \times 10^8$ and $Pr = 6.26$. The short vertical dotted line corresponds to the approximate location of the bifurcation point at $1/Ro_c$.

$1/Ro$. Qualitatively, the strong increase at modest rotation rates has been observed by several investigators, including Rossby (1969), Zhong et al. (1993), Liu & Ecke (1997b), Liu & Ecke (2009), King et al. (2009). It is understood to be caused by the Ekman vortices (Hart 1995, 2000; Julien et al. 1996b; Hart et al. 2002; Kunnen et al. 2006; Zhong et al. 2009; Stevens et al. 2010b) that form in the presence of the Coriolis force; these vortices extract additional warm (cold) fluid from the bottom (top) thermal BL. Since the enhancement of $Nu$ started discontinuously at $1/Ro_c$, we are led to conclude that the Ekman vortices formed only above this threshold. With further increase of $1/Ro$ one sees a dramatic decrease of $Nu$. In this range, the depression of $Nu$ due to the TP effect was stronger than the Ekman enhancement, and $Nu$ reached values well below those without rotation. Of course, it is known that sufficiently large rotation rates will suppress the convective heat transport completely, leading to $Nu = 1$ when the rotation has increased the critical Rayleigh number $Ra_{c}(\Omega)$ (Chandrasekhar 1981) up to the value of $Ra$ for the particular run. The transition from the turbulent-convection regime to the regime near the onset of convection was considered in some detail recently by King et al. (2009).

3.2.2. The range $1/Ro < 1/Ro_c$

In figure 5, we show data for $1/Ro < 0.7$ for all runs that resolved this range. The labels in each panel give, from left to right, the run number, $Ra/10^9$, and $Pr$ (see also table 1). The bifurcation at $1/Ro_c$ is apparent for several of the runs. However, especially from the data at relatively small $Pr$ and large $Ra$, one sees that the rotation already has an influence on $Nu$ in the range below $1/Ro_c$. Runs E7, E8 and E9 clearly show that there is an initial small enhancement, followed by a small depression, of $Nu$. For all cases studied in this work, these effects are less than 1%. The results for the time-averaged and azimuthally averaged thermal gradient, and of the thermal amplitude of the LSC, to be presented in §§4 and 5.2, respectively, likewise indicate that interesting modifications of the system occur already in this range below $1/Ro_c$. To our knowledge the origin of these phenomena is not clear at this time, although one might expect that in part they could be explained by considerations involving
centripetal forces and differential rotation at different heights in the sample such as those pursued recently by Stevens et al. (2010a). It seems unlikely that TP suppression of Nu would play any role in this range of $1/Ro$.

3.2.3. The dependence of $1/Ro_c$ on Ra and Pr

In our work, the bifurcation at $1/Ro_c$ always occurred at relatively small rotation rates, and for that reason, was difficult to determine with high accuracy. Nonetheless, we were able to obtain the data shown in figure 6 as a function of Ra (figure 6a) and of Pr (figure 6b). We obtained $1/Ro_c$ from the intercept of two straight lines. For the runs where adequate measurements existed, one line was placed through the data just below and the other through the data just above the transition. The results do not resolve any significant dependence on Ra, but do show a significant dependence on Pr. The Pr-dependence can be represented by an effective power law $1/Ro_c = a Pr^\zeta$, with $a = 0.77 \pm 0.05$ and $\zeta = -0.42 \pm 0.04$, which is shown as the solid line in the figure. We do not necessarily expect this relation to be valid much beyond the quite narrow Pr range of our data. Thus, for instance, it is known from DNS (Zhong et al. 2009; Stevens et al. 2010a) that there is no enhancement of Nu, and most likely no bifurcation, for $Pr \lesssim 1$ and $Ra$ near $10^8$ or less. It was also shown by experiment on a sample with $\Gamma = 0.50$ (Niemela et al. 2010) at the very large $Ra \approx 4 \times 10^{15}$ and $Pr$ near six that Nu decreases with increasing $1/Ro$ as soon as

![Figure 5](image-url)
rotation starts, implying that there is no bifurcation at a finite 1/Ro_c. Understanding the origin of the Pr-dependence, and indeed of the very existence, of the bifurcation point in our parameter range remains an interesting challenge.

3.2.4. The dependence of the initial slope S_1 of Nu(Ω)/Nu(0) on Ra and Pr

One can see from the data in figure 5 and just above 1/Ro_c that Nu increased linearly beyond the bifurcation point. This increase was attributed (Zhong et al. 2009) to Ekman pumping. However, for 1/Ro > 1/Ro_c we expect two competing effects. Ekman pumping will enhance and the TP effect will diminish the heat transport. Without theoretical guidance it is not possible to rigorously separate these two phenomena in a quantitative analysis of the data. Consistent with the data, we shall assume that the Ekman pumping is responsible for the linear rise of Nu above the bifurcation, and that the TP suppression is of quadratic or higher order so that it comes into play only as 1/Ro becomes larger. Of course, we cannot rule out that the linear rise is the net result of a positive contribution from Ekman pumping and a smaller but negative contribution from the TP effect.

We fit the available data above but close to 1/Ro_c to a straight line. This yielded the initial slope S_1 of Nu(Ω)/Nu(0) = S_0 + S_1 · (1/Ro) + ···. The results for S_1 are shown in figure 7 as a function of Ra (figure 7a) and of Pr (figure 7b). As was the case for 1/Ro_c, the measurements of S_1 do not reveal any dependence on Ra, except that the point at the largest Ra (run E8, ΔT = 31.7 K, see table 1) falls well below the other three at the same Pr. Because of the exceptionally large value of ΔT for this point, we expect that the low value may be attributable to non-Boussinesq effects (Ahlers et al. 2006a). However, the recent work by Niemela et al. (2010) for the very large Ra = 4.3 × 10^{15}, Γ = 0.50 and Pr = 5.9 showed that Nu(Ω) ∝ (1/Ro)^{-0.024}, corresponding to a depression of Nu due to rotation. It is possible that our low value of S_1 for our largest Ra is due to this observed decrease of the heat transport by the Ekman-pumping effect with increasing Ra. Thus, it would be interesting to obtain strictly Boussinesq data over the Ra range from 10^{11} to 10^{15}. 

![Figure 6](image-url)
It is already apparent from figure 4 that $S_1$ does not depend strongly on $Pr$. However, as was the case for $1/Ro_c$, at a quantitative level there is a modest increase of $S_1$ with $Pr$ (see figure 7b) over our parameter range. The solid line corresponds to $S_1 = S_{1,0} Pr^\eta$ with $S_{1,0} = 0.058 \pm 0.002$ and $\eta = 0.27 \pm 0.02$. Although this power law may provide a useful representation of the data, we again do not expect it to hold necessarily much beyond the data range. We note again that, at least for $Ra = O(10^8)$, DNS has indicated (Zhong et al. 2009) that there is no positive enhancement of $Nu$ for $Pr \lesssim 1$; thus our empirical fit, which yields a positive slope $S_1 = 0.058$ for $Pr = 1$, ceases to be meaningful in that range. We note that the heat transport by Ekman pumping (which we assumed to be exclusively responsible for a positive slope $S_1$) increases as $Pr$ increases, as previously reported. However, the effect is modest; $S_1$ increased by less than 20% as $Pr$ changed by about a factor of two. Again, as seen in figure 4 and as will be discussed in the next section, the main change with decreasing $Pr$ and increasing $Ra$ is an enhancement of the suppression of $Nu$ below the initial increase attributed to Ekman pumping. We attribute this suppression to the TP effect and see that it leads to a strong $Pr$-dependence at intermediate to large $1/Ro$.

3.2.5. The range of $1/Ro$ well above the bifurcation

As seen already in figure 4, $Nu$ initially increases due to the Ekman pumping, but then decreases again due to the TP effect. In figure 8, we show more data for other $Pr$ and $Ra$ values. Figure 8(a) gives the results for the three values $Pr = 3.05, 4.38$ and $6.42$ at the same $Ra = 1.2 \times 10^9$. At (not too small) constant $1/Ro$ one sees a dramatic increase of $Nu(\Omega)/Nu(0)$ with increasing $Pr$. Some of this increase comes from an increase in the efficiency of the heat transport by the Ekman-pumping effect. However, we see from figure 7 that the increase in $S_1$ over this range of $Pr$ is less than 20%, while the effect of $Pr$ on $Nu$ seen in figure 8(a) is much larger. We conclude that the depression of $Nu$ due to the TP effect depends strongly on $Pr$ and is larger at smaller $Pr$. 

Figure 7. (a) The initial slope $S_1$ of $Nu(\Omega)/Nu(0) = b + S_1 \cdot (1/Ro) + \cdots$ as a function of $Ra$. (b) The initial slope $S_1$ as a function of $Pr$. The scales of both axes are logarithmic. Open circles, $Pr = 6.26$; solid circles, $Pr = 4.38$; open squares, $Pr = 3.62$; solid squares, $Pr = 3.05$. The solid line is a power law with an exponent $\eta = 0.27$. 

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Let us consider the decrease of $Nu(Ω)/Nu(0)$ with decreasing $Pr$ along a vertical line in figure 8(a), i.e. along a line of constant (not too small) $1/Ro$ and at constant $Ra$. We note that along that line the inverse Ekman number $1/Ek$ increases when $Pr$ decreases (see also table 1), indicating that the Coriolis force (which generates the TP effect) increases relative to the viscous force. Thus, as seen in the experiment, one would expect the TP effect (which is inhibited by viscous dissipation) to become stronger as $Pr$ decreases; but of course we do not have a quantitative theory.

Figure 8(b) shows results for $Nu(Ω)/Nu(0)$ for several values of $Ra$ at constant $Pr = 4.38$. In §3.2.4, we saw that the Ekman-pumping enhancement of $Nu$ just above $1/Ro_c$ was independent of $Ra$ within our resolution. However, the data suggest that the TP depression depends strongly on $Ra$, and increases as $Ra$ increases. For $Ra$ near $4 \times 10^{15}$ we learned from the measurements of Niemela et al. (2010) for $\Gamma = 0.5$ that this depression eventually will become strong enough to completely eliminate any enhancement, leading to a depression of $Nu$ that starts as soon as $1/Ro$ exceeds zero.

3.3. The Nusselt number as a function of the Taylor number

It is useful to see the relationship of our measurements to some other results in the literature. In figure 9(a), we show the location of our data points for $Pr = 4.38$ in the $Ta$–$Ra$ parameter space (the data at other $Pr$ cover similar ranges). Our measurements were made at constant applied temperature difference, varying the rotation rate $Ω$, i.e. at constant $Ra$ as a function of $Ta$ or $Ek$ and of $Ro$. Thus they fall along horizontal lines in the figure. For our data, the locus $Ro = 1$ is shown by the dotted line. Often measurements by others were made at constant $Ω$ (i.e. constant $Ta$ or $Ek$), varying the applied temperature difference (i.e. $Ra$ and $Ro$). Those data then would fall along vertical lines in figure 9(a). The results of Liu & Ecke (2009) fall approximately into the rectangle in the lower left of the figure. Those of King et al. (2009) are in the range below the dashed line. The heavier solid line in the
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Figure 9. (Colour online) (a) Location of the measurements for \( Pr = 4.38 \) in the \( Ra–Ta \) parameter space. The dotted (dash-dotted) line corresponds to \( 1/Ro = 1 \) \( (1/Ro = 10) \) for our data. The rectangle in the lower left indicates the approximate range covered by the data of Liu & Ecke (2009). The data of King et al. (2009) are for parameters in the range below the dashed line. The thicker solid line is the location of the critical Rayleigh number for the onset of convection. Solid circles (red online): run E4. Open circles (red online), run E5. Solid squares (blue online), run E6. Open squares (blue online), run E7. Solid diamonds (purple online), run E8. (b) The reduced Nusselt number \( Nu/Ra^{0.304} \) as a function of the Taylor number. The symbols are as in (a). Results without rotation \( (Ta = 0) \) depend slightly on \( Ra \) (see figure 3) but fall between the two horizontal lines.

Figure is the location of the onset of convection at \( Ra = Ra_c(Ta) \) (Chandrasekhar 1981).

In figure 9(b), we show our results for the reduced Nusselt number \( Nu/Ra^{0.304} \) as a function of \( Ta \) for the \( Ra \) values explored for \( Pr = 4.38 \). The two thin horizontal lines indicate the range covered by \( Nu/Ra^{0.304} \) in the absence of rotation (see figure 3). The discontinuous onset of the Nusselt enhancement due to Ekman pumping can also be seen in this representation. However, the critical value \( Ta_c \) for that onset depends strongly upon \( Ra \), whereas \( 1/Ro_c \) at constant \( Pr \) had been found to be independent of \( Ra \) (see figure 6). Below that onset the rotation had only a minor influence on the heat transport, as seen from the fact that the data fall more or less between the two horizontal lines. As \( Ta \) increases, the Ekman enhancement is followed by the TP suppression, with the suppression generally being stronger as the data points approach the bifurcation line \( Ra_c(Ta) \) more closely. The bifurcation line can be regarded as the locus of complete TP domination, where the Nusselt number is suppressed all the way to \( Nu = 1 \). In the range of very strong TP suppression, only somewhat above the convective onset, some have suggested that \( Nu \sim Ra^{6/5} \) (King et al. 2009); but the range over which such a power law represents the data is quite narrow and thus it seems questionable to us whether it is useful to describe \( Nu(Ra) \) in this way. In any case, any extrapolation of such a power law beyond the range of the data from which it was derived is unjustified.
Figure 10. Normalized vertical temperature variation $\Delta T_w(1/Ro)/\Delta T_w(0)$ along the sidewall. (a) Results for $Ra = 1.2 \times 10^9$. Solid squares: $Pr = 3.05$ (E10, $T_m = 60.0\,^\circ C$, $\Delta T = 1.21\, K$). Open diamonds: $Pr = 4.38$ (E5, $T_m = 40.0\,^\circ C$, $\Delta T = 1.99\, K$). Solid downward-pointing triangles: $Pr = 6.42$ (E1, $T_m = 23.0\,^\circ C$, $\Delta T = 4.68\, K$). (b) Results for $Pr = 4.38$ ($T_m = 40.00\, ^\circ C$). Solid upward-pointing triangles: $Ra = 1.8 \times 10^{10}$ (E8, $\Delta T = 31.69\, K$). Open squares: $Ra = 8.9 \times 10^9$ (E7, $\Delta T = 3.98\, K$). Open diamonds: $Ra = 1.2 \times 10^9$ (E5, $\Delta T = 1.99\, K$).

4. Temperature gradients near the sidewall

Without rotation and away from the thermal BLs the time-averaged temperature in a cylindrical RBC system is known to be nearly uniform along the vertical centreline. Depending on $Pr$, the small prevailing temperature gradient can be either stabilizing or destabilizing (Tilgner, Belmonte & Libchaber 1993; Brown & Ahlers 2007b). The situation is different near the sidewall. Plumes emitted by the top and bottom thermal BLs tend to be swept by the LSC and to travel upwards mostly near the wall. This leads to a significant destabilizing vertical gradient which, depending on $Ra$ and $Pr$, can be 10% or more of $\Delta T/L$ (Brown & Ahlers 2007b). DNS showed that rotation leads to a much larger destabilizing temperature gradient along the centreline (Julien et al. 1996b; Hart & Olsen 1999; Kunnen et al. 2010). This effect was attributed to diminished vertical and enhanced lateral mixing in the presence of rotation (Julien et al. 1996b). Here we report on the effect of rotation on the temperature gradient near the wall.

The fits of (2.1) to the three sets of sidewall temperatures gave the azimuthally averaged mean temperatures $T_{w,m}, T_{w,t}$ and $T_{w,b}$ along the wall at the three vertical positions $L=0, L/4$ and $-L/4$, respectively. These in turn yielded the vertical temperature variation along the wall and away from the top and bottom BLs.

For $\Omega = 0$ the estimate $\Delta T_w/\Delta T = 2 \times \langle T_{w,b} - T_{w,t} \rangle_t/\Delta T$ of the time-averaged vertical temperature variation along the entire sample (excluding the BLs) was reported by Brown & Ahlers (2007b) for a sample of the same size and shape as the one used in the present work. Here we show $\Delta T_w(1/Ro)/\Delta T_w(0)$ in figure 10 (results for $\Delta T_w/\Delta T$ for run E7 were reported by Stevens et al. 2009). In figure 10(a), results at constant $Ra = 1.2 \times 10^9$ are shown for three values of $Pr$. At small $Pr \simeq 3$ and over a wide range of $1/Ro$ one sees a dramatic enhancement of the temperature variation. As $Pr$ increases, this effect becomes smaller. Figure 10(b) shows the results at constant $Pr$ as a function of $Ra$. Again, the enhancement is evident, with a larger effect observed at the larger $Ra$ values.
At small $1/Ro$ there is interesting structure in this variable, as shown in figure 11 for $Pr = 4.38$ and several $Ra$. One sees that there is an initial decrease in the temperature gradient with increasing rotation rate, followed by an increase above a characteristic value of $1/Ro$. It is interesting to note that the relatively sudden change from a decreasing to an increasing gradient occurs at a value of $1/Ro$ that is below rather than at the $1/Ro_c$ determined from the Nusselt-number measurements and shown by the short vertical lines in the figure.

The overall increase, over a wide range of $1/Ro$, of $\Delta T_w$ can be attributed qualitatively to enhanced plume detachment from the BLs, which leads to a greater plume density near the sidewall, as well as to enhanced Ekman-vortex activity. This phenomenon is evident in the recent DNS of Zhong et al. (2009). The initial decrease of $\Delta T_w$ at small rotation rates is accompanied by an enhanced strength of the LSC, as we shall see below in § 5.2.

5. The large-scale circulation

5.1. Reynolds numbers of the large-scale circulation

For cylindrical samples with $\Gamma \simeq 1$ and $\Omega = 0$ there have been numerous measurements of the frequencies or periods of characteristic oscillatory modes of the system by several methods. Single-point temperature and velocity determinations have been used, and auto-correlation functions as well as cross-correlation functions of temperature time series at various locations in the sample have been determined. In most if not all of these measurements, it is now understood that the detected oscillations are caused by a torsional (Funfschilling & Ahlers 2004; Funfschilling et al. 2008) or sloshing (Brown & Ahlers 2009; Xi et al. 2009; Zhou et al. 2009) mode of the LSC dynamics with a frequency that is synchronous with the LSC turnover frequency, provided $Ra$ is not too large (Brown et al. 2007); at most locations in the sample the torsional mode leads to synchronous temperature and velocity oscillations. This was discussed recently in some detail by Brown et al. (2007) (see also Xia 2007), who also give extensive references to the literature. Here we cite only the particularly noteworthy work of Qiu & Tong (2002), who showed that velocity- and temperature-oscillation measurements on the same sample do indeed
yield the same frequencies. The large body of previous work shows that any one of the above-mentioned measurements can be used to compute a Reynolds number \( Re \) of the LSC, provided \( Ra \) is less than some characteristic \( Ra^* \simeq 3 \times 10^9 \). Even for \( Ra > Ra^* \), differences between \( Re \) determined by different methods are not very large. Since we are concerned here with the \( 1/Ro \)-dependence of \( Re \), we shall focus on \( Re(\Omega)/Re(0) \), which we assume to be insensitive to the measurement that was done even for \( Ra > Ra^* \).

In the present paper, we determined \( Re \) from auto-correlation functions of the eight sidewall temperatures \( T_{m,i} \) at the middle \((z = 0)\) location
\[
\tilde{C}^{m,m}_{T_i}(\tau) = \langle [T_{m,i}(t) - \langle T_{m,i}\rangle_t] \times [T_{m,i}(t + \tau) - \langle T_{m,i}\rangle_t] \rangle_t, \quad (5.1)
\]
separately at each azimuthal location \( i \). We normalized these correlation functions to form
\[
C^{m,m}_{T_i}(\tau) = \frac{\tilde{C}^{m,m}_{T_i}(\tau)}{\tilde{C}^{m,m}_{T_i}(0)}. \quad (5.2)
\]
Finally, we computed the average \( C^{m,m}_{T_i}(\tau) \) of \( C^{m,m}_{T_i}(\tau) \) over all eight azimuthal locations \( i = 0, \ldots, 7 \).

Figure 12(a) shows the averaged auto-correlation function \( C^{m,m}_T(\tau) \). The overall decay is obvious; but close inspection also reveals an oscillatory component. To make the oscillations more dominant, we took the second derivative with respect to \( \tau \), which yielded the result shown in figure 12(b). This result could be fitted fairly well
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Figure 13. (Colour online) Some properties of the LSC for $Ra = 2.25 \times 10^9$, $Pr = 4.38$ (run E6, $\Delta T = 3.98\, K$) as a function of $1/Ro$ on a logarithmic scale. Solid symbols: the time-averaged LSC amplitudes $\langle \delta_k \rangle$ for $k = m$ (circles, green) and $k = t$ (squares, blue). Open symbols: the root-mean-square deviations $\langle \delta T_k \rangle$ from the fits of (2.1) for $k = m$ (circles, green) and $k = t$ (squares, blue). Stars and pluses: the time averages of the probable errors of $\delta_k$ for $k = m$ (stars, green) and $k = t$ (pluses, blue). The vertical dotted line corresponds to $1/Ro_{c}$ as determined from the Nusselt-number measurements.

by the function

$$C_{fit} = c_1 \exp\left(-\tau/\tau^{bg}\right) \cos\left(2\pi * \tau/\tau^{ac}\right).$$

This fit yielded $\tau^{ac} = 104.7 \pm 0.7$ for this particular case. We took the ratio $\tau^{ac}(\Omega)/\tau^{ac}(0)$ to be equal to $Re(\Omega)/Re(0)$. Results for this ratio are shown in figure 12(c) for $Pr = 4.38$ and three different values of $Ra$. Quite remarkably, slow rotation, up to $1/Ro \approx 0.3$ or so, had little or no effect upon $Re$. This differs from the $1/Ro$-dependence of the temperature gradient near the sidewall (see §4 and figure 11), which decreased at much smaller $1/Ro$, and the LSC temperature amplitude $\langle \delta_k \rangle$ (see §5.2 and figure 13) which increased as soon as rotation was started. However, for $1/Ro \gtrsim 0.3$ $Re$ did decrease. This decrease started well below $1/Ro_c \approx 0.41$ as determined from the Nusselt-number measurements (see §3.2.3 and figure 6). The decrease of $Re(\Omega)/Re(0)$ seemed to start at a smaller value of $1/Ro$ for the larger $Ra$. We were unable to measure $Re(1/Ro)$ for larger $1/Ro$ because we did not observe well-defined oscillations in the correlation functions. However, an explanation of the various $1/Ro$-dependences at small $1/Ro$ will already be an interesting challenge.

5.2. Strength of the large-scale circulation in the presence of rotation

It is generally believed that the formation of the Ekman-vortex structures diminishes the strength of the LSC, and that for $1/Ro$ significantly above unity, the LSC ceases to exist (see e.g. Hart et al. 2002 and Kunnen et al. 2008b). Here we report on the $1/Ro$-dependences of the temperature amplitudes $\delta_k$, $k = m, t$ of the LSC that were obtained by fitting (2.1) to the sidewall-temperature measurements at the two vertical levels $z = 0$ ($k = m$) and $z = L/4$ ($k = t$) as described in §2.5. We find that the $\langle \delta_k \rangle$ initially increase, and then decrease, with increasing $1/Ro$. Near $1/Ro \approx 1$ they become comparable to the root-mean-square temperature fluctuations about the
fit and a more detailed analysis would be required to determine whether the LSC continues to exist in the presence of these large fluctuations.

In figure 13, we show the time-averaged LSC temperature amplitudes \( \langle \delta_k \rangle, k=m,t \), as solid symbols as a function of \( 1/Ro \) on a logarithmic scale. Similar data, for run E7, were shown by Stevens et al. (2009). The dotted vertical line corresponds to \( 1/Ro_c \) as determined from the Nusselt-number measurements. At very small \( 1/Ro \), the amplitudes at \( z=0 \) and \( z=L/4 \) differ somewhat from each other as had been noted before (Brown & Ahlers 2007b); but with increasing rotation rate both increase and approach each other. The amplitudes merge near a maximum that is located somewhat below \( 1/Ro_c \) and then decrease with increasing \( 1/Ro \). The origin of the initial increase of the \( \langle \delta_k \rangle \) is not obvious, but may be associated with the centripetal effects discussed by Stevens, Clercx & Lohse (2010a). Similarly, it is unclear why the maximum of \( \langle \delta_k \rangle \) should be located well below \( 1/Ro_c \). With increasing rotation rate beyond the one of the maximum the amplitudes dropped to about 40\% of their largest value, presumably because of the interactions with the Ekman-vortex structure that is expected to form starting at \( 1/Ro_c \). For \( 1/Ro \gtrsim 0.7 \) the amplitudes remained more or less constant as \( 1/Ro \) was increased.

Also shown in the figure are the root-mean-square amplitudes of the temperature fluctuations about the cosine fit. Interestingly, there was an apparently sudden onset of a mild increase which coincided with \( 1/Ro_c \). For larger rotation rates the fluctuations grew somewhat and near \( 1/Ro = 0.7 \) reached values close to the amplitudes \( \langle \delta_k \rangle \). Beyond that point it is difficult for us to be sure about the existence of an LSC without more detailed analysis, even though the time averages of the standard errors of the \( \delta_k \) (also shown in the figure) remained well below the \( \delta_k \) themselves.

5.3. Time-dependence of the large-scale circulation temperature amplitude

In figure 14(a), we show a time series of \( \delta_m \) from run E8 in the absence of rotation. One sees that \( \delta_m \) fluctuates as seen before, and that on rare occasions it briefly vanishes (Brown et al. 2005a; Brown & Ahlers 2006b). These events, where the flow briefly stops, are known as cessations and have been the subject of extensive experimental.
Figure 15. (a) The temperature amplitude $\delta_m$ at the horizontal mid-plane for $Ra = 1.8 \times 10^{10}$, $Pr = 4.38$ and $1/Ro = 0.432$ (run E8) as a function of time. (b) The deviation of the azimuthal orientation $\theta_m - \theta_{\text{fit}}$ from a straight-line fit $\theta_{\text{fit}}(t)$ corresponding to (a). The vertical lines are guides to the eye to show the correlation between strong dips in $\delta_m$ and sudden jumps in $\theta_m - \theta_{\text{fit}}$. (c) and (d) are detailed illustrations of a cessation for $Ra = 1.8 \times 10^{10}$, $Pr = 4.38$ and $1/Ro = 0.360$.

and theoretical studies. They occur approximately once a day. In order to define a cessation, we require that $\delta_m$ decreases below a critical value $\delta_l$. Somewhat arbitrarily $\delta_l$ was chosen to be $0.15\langle \delta_m \rangle$ (see e.g. Brown & Ahlers 2006b for a discussion of the choice and influence of $\delta_l$); it is indicated by the horizontal dashed line in the figure.

In figure 14(b), we show similar data for $\delta_m$, but for the rotating system with $1/Ro = 0.26$, which is still well below the critical value $1/Ro_c \simeq 0.41$. It is immediately clear that the frequency of cessations increased dramatically due to the rotation.

In order to show that the vanishing of the LSC amplitude $\delta_m$ is accompanied by the sudden change in the LSC orientation that is typical of cessations, we show another data set in figure 15. Here we compare $\delta_m$ (figure 15a) with the LSC orientation (figure 15b). However, in the rotating system the average retrograde rotation, to be discussed in §5.4, had to be subtracted before the dynamics of the LSC orientation could be seen. Thus we fit a straight line to $\theta_m(t)$ over a long time interval to obtain $\theta_{\text{fit}}$, and plotted $\theta_m - \theta_{\text{fit}}$ as a function of time in figure 15(b). The vertical lines are guides to the eye, and show that near-discontinuous jumps in the LSC orientation coincided with near-vanishing of $\delta_m$. The jumps in orientation seem to be of randomly different size as found in the absence of rotation, but we have not done a quantitative statistical analysis. In figures 15(c) and 15(d), we show details of a typical cessation. As expected, the amplitude decreases dramatically, and when it is near its minimum, the LSC orientation changes quite suddenly. We conclude that the qualitative features
of cessations in the presence of rotation are the same as they are for $1/R_o = 0$ (Brown & Ahlers 2006b).

In figure 16, we show the average number of cessations $\alpha$ observed at various values of $1/R_o$. One sees that $\alpha$ increased dramatically with $1/R_o$, from near $10^{-5}$ close to $1/R_o = 0$ to just above $10^{-4}$ near $1/R_o = 0.2$. Apparently, there was then a plateau which extended up to $1/R_o \simeq 0.4$. At $1/R_o$ there was a jump in $\alpha$ by a factor of three, followed by a further gradual increase up to a frequency of about once every 500 s near $1/R_o = 0.6$. For even larger $1/R_o$ cessations were so frequent that it became difficult to identify individual events.

5.4. Retrograde rotation of the large-scale circulation

The LSC takes place in a near-vertical circulation plane. In the absence of viscous drag, one would expect the orientation of this plane to remain constant in an inertial frame because the LSC has angular momentum (Brown & Ahlers 2006a). Consequently, one would expect the orientation $\theta_0$ in the frame of a sample rotating with angular velocity $\Omega$ to undergo retrograde rotation at an angular velocity $\omega = -\Omega$. This will be altered by the presence of viscous drag on the walls across the viscous boundary layers, so that in the physical system $|\omega| < |\Omega|$. This phenomenon is contained in a deterministic model by Hart et al. (2002), and in more detail in a stochastic model by Brown & Ahlers (2006a) that was developed to explain the influence of Earth’s Coriolis force on the statistical properties of the LSC. However, measurements by both groups, as well as more recent ones by Kunnen et al. (2008b), showed that estimates of the drag by the walls based on the ordinary shear viscosity of the fluid were too small and gave values of $-\omega/\Omega$ that were too large (albeit of course less than one). Hart et al. (2002) introduced a turbulent viscosity that led to values of $-\omega/\Omega$ that were closer to the experiment.

In figure 17, we show some of our measurements of the orientation $\theta_0$ of the LSC circulation plane as a function of time. The numbers in the figures are the value of $1/R_o$. For $1/R_o = 0$ the data reveal the usual azimuthal diffusion that has been observed before. For $1/R_o > 0$ the retrograde rotation is obvious and on average led to a linear decrease of $\theta_0$ with time. Measurements of $\theta_0(t)$ for $1/R_o > 0$ were also made by Hart et al. (2002) and by Kunnen et al. (2008b). The latter authors found that $\theta(t)$ ceased to follow a linear dependence when $1/R_o \gtrsim 0.2$ or 0.3. They interpreted this to imply that the LSC gradually disappeared as $1/R_o$ increased. Our data continued
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Figure 17. (Colour online) The orientation $\theta_m$ of the LSC circulation plane as a function of time. The numbers in the figures indicate the values of $1/Ro$. All data are for run E8 with $Ra = 1.8 \times 10^{10}$ and $Pr = 4.38$.

Figure 18. (a) Retrograde rotation velocity $-\omega$ as function of the sample rotation velocity $\Omega$. (b) The ratio $-\omega/\Omega$ as a function of $\Omega$. The data are for run E8 ($Ra = 1.8 \times 10^{10}$, $Pr = 4.38$).

to reveal retrograde rotation up to $1/Ro \simeq 1$. However, for $1/Ro \gtrsim 0.4 \simeq 1/Ro_c$ (see §3.2.3) the time traces became noisier; but there remained linear trends on average over sufficiently long time intervals as shown by the data in figure 17 that extend up to $1/Ro = 1.1$.

Straight-line fits to data such as those in figure 17 led to the retrograde rotation rates $\omega$ that are shown in figure 18(a) as a function of $\Omega$ and in figure 19(a) and c as a function of $1/Ro$. The above-mentioned models (Hart et al. 2002; Brown & Ahlers 2006a) predicted that $\omega$ should be proportional to $\Omega$, i.e. $-\omega/\Omega$ should be independent of $\Omega$ or $1/Ro$. They also yielded only a very weak dependence on $Ra$, and predicted a value only somewhat less than unity for $-\omega/\Omega$. For example, the model of Brown & Ahlers (2006a), which assumed viscous drag by the sidewalls based on the ordinary shear viscosity of the fluid, predicted $-\omega/\Omega = 0.77$, 0.83 and 0.85 for runs E6, E7 and E8, respectively. For small rotation rates a linear dependence of $\omega$ on $\Omega$ was indeed observed within their experimental resolution by Hart et al. (2002). A glance at figure 18(a) suggests that a linear dependence is also consistent with our data; but closer inspection reveals a more complex behaviour. This is seen in figure 18(b) and in figure 19(b, d), where we show the measured ratios. The data show interesting structure even for $1/Ro < 1/Ro_c$. We note that the most rapid variation with $1/Ro$ occurs in
Figure 19. (a, c) Retrograde rotation velocity $-\omega$ as function of the inverse Rossby number $1/\Ro$. (b, d) The ratio $-\omega/\Omega$ as a function of $1/\Ro$. The panels (a) and (b) show the dependence on $Ra$ at nearly constant $Pr$, and the bottom ones give the dependence on $Pr$ at nearly constant $Ra$. Open circles, run E8 ($Ra = 1.8 \times 10^{10}$, $Pr = 4.38$). Solid squares, run E7 ($Ra = 9.0 \times 10^{9}$, $Pr = 4.38$). Open triangles, run E6 ($Ra = 2.2 \times 10^{9}$, $Pr = 4.38$). Solid diamonds, run E11 ($Ra = 8.0 \times 10^{9}$, $Pr = 3.05$). Open diamonds, run E3 ($Ra = 2.2 \times 10^{9}$, $Pr = 6.26$). The vertical dotted lines indicate the location of $1/\Ro_c$ as determined from the Nusselt-number measurements.

6. Summary and discussion

A major emphasis of much of the work on rotating turbulent RBC has been on the global heat transport. As reported briefly elsewhere (Zhong et al. 2009) and found by several other authors (Rossby 1969; Zhong et al. 1993; Liu & Ecke 1997b; Kunnen et al. 2006, 2008b; Liu & Ecke 2009), we found that rotation at modest rates can enhance the heat transport (see figures 4, 5 and 8), rather than suppressing it as expected on the basis of the TP effect and as seen close to the onset of convection (Chandrasekhar 1981). Over our parameter range we found that this enhancement can be up to about 20%, but the DNS of Stevens et al. (2010b) showed that it can be as large as 30% when $Ra$ is near $10^8$ and $Pr$ is near 20 or so. The enhancement is now understood to be caused by Ekman vortices that form due to the
rotation (Boubnov & Golitsyn 1986, 1990; Hart 1995, 2000; Julien et al. 1996b, 1999; Vorobieff & Ecke 1998, 2002; Hart et al. 2002; Kunnen et al. 2006; Zhong et al. 2009; Stevens 2010b); the vortex cores extract additional fluid out of the thermal BLs and thereby enhance the heat transport. We presented quantitative measurements of this effect as a function of $Pr$ and $Ra$. It should be noted that there are also experiments (Pfotenhauer et al. 1984; Niemela et al. 2010) that, for certain $\Gamma$, $Pr$ and $Ra$, did not reveal any enhancement of $Nu$. Thus it remains to understand in detail the parameter ranges over which enhancement occurs.

Quite remarkably, the enhancement of $Nu$ did not start at $1/Ro = 0$, but rather occurred only above a finite threshold rotation rate $1/Ro_c$ (Stevens et al. 2009) (see figures 4 and 5). This threshold decreased mildly with increasing $Pr$, but seemed essentially independent of $Ra$ over our parameter range (see figure 6). At the threshold the enhancement of $Nu$ started continuously from zero, consistent with a supercritical bifurcation. Bifurcations between turbulent states are somewhat unexpected because the highly fluctuating systems are presumed to already sample all of the available phase space; when bifurcations do occur, then they are most likely associated either with changes in the boundary conditions (or BLs) or in relevant large-scale structures. Consistent with these general considerations, we interpret the observed bifurcation as a transition from a turbulent state without to one with Ekman pumping, i.e. to a discontinuous onset of the formation of large-scale structures in the form of Ekman vortices; but we do not understand why that formation should have a finite threshold.

Close to but above $1/Ro_c$ the Ekman-pumping enhancement of $Nu$, as measured by the initial slope $S_1$ of $Nu(1/Ro)$ just above $1/Ro_c$, became larger with increasing $Pr$, but only by just under 20% over our $Pr$ range from 3.0 to 6.4 (see figure 7). The data did not reveal any significant dependence of $S_1$ on $Ra$. The smaller enhancement at smaller $Pr$ was attributed by Zhong et al. (2009) to the larger thermal diffusivity, which promotes lateral heat exchange and thereby tends to reduce the pumping efficiency of the Ekman vortices. It was suggested by Stevens et al. (2010b) that, with increasing $Pr$, there is a $Pr$ value of maximum enhancement because at very large $Pr$ the viscous BLs become so thick that the Ekman vortices no longer have access to the thermal BLs, where the enhanced heat transport originates. Our measurements did not extend to this large $Pr$ regime, and experiments or DNS that reveal a maximum of $S_1$ as a function of $Pr$ at constant $Ra$, to our knowledge, have not yet been performed. However, at constant $1/Ro = 1$, where one would expect the TP suppression to still be very small, Stevens et al. (2010b) showed from DNS that $Nu(1/Ro)/Nu(0)$ has a maximum near $Pr = 10$.

We do not know of a rigorous way to separate the Ekman enhancement and the TP suppression of the Nusselt number; but the data do suggest a natural separation. One finds that the curves for $Nu(1/Ro)/Nu(0)$ versus $1/Ro$ start out with a common $Ra$ independent but weakly $Pr$-dependent monotonically increasing envelope, but then fall away to lower values at a dramatic rate, starting at different values of $1/Ro$ that depend on $Ra$ and $Pr$. This is illustrated well in figures 4 and 5, and we demonstrate it further in figure 20, which shows all of our data. There the solid line is an empirical power-law fit to the data from run E2 for $0.5 \leq 1/Ro \leq 4$. Similar envelopes could be drawn for each $Pr$, and would differ only slightly from the one shown in the figure. The data thus suggest that their natural interpretation is to attribute the common envelope to an only weakly $Pr$-dependent and $Ra$-independent Ekman enhancement, and the depression below it to a strongly $Pr$- and $Ra$-dependent TP suppression. This interpretation implies that at constant $Ra$ the depression becomes more severe as the Prandtl number decreases, and that at constant $Pr$ it becomes larger at larger
Figure 20. (Colour online) All of the data for $y = Nu(1/Ro)/Nu(0)$ as a function of $x = 1/Ro$ on linear scales. Solid black circles: E1; solid red circles: E2; solid blue circles: E3; solid purple circles: E4; open black circles: E5; open red circles: E6; open blue circles: E7; open purple circles: E8; solid black diamonds: E9; solid red diamonds: E10; solid blue diamonds: E11. The solid line is the power law $y = y_0 + m(x - x_0)^\zeta$ with $y_0 = 1$, $x_0 = 0.4$, $m = 0.074$ and $\zeta = 0.66$ obtained from fitting the E2 data for $0.5 \lesssim 1/Ro \lesssim 4$.

$Ra$. It is then the competition between this TP reduction and the Ekman enhancement that determines the strong $Pr$- and $Ra$-dependence of the $Nu$ enhancement near the maximum of $Nu(1/Ro)$. An alternative to this interpretation would imply that the Ekman enhancement becomes less efficient as $1/Ro$ increases, thus leading to the maximum in the data. This alternative seems unlikely to us.

Qualitatively we can understand the strong $Pr$-dependence at constant $Ra$ and $1/Ro$ displayed in figure 8(a) by considering the variation of the Ekman number $Ek$, which represents the ratio of the strength of the viscous force to that of the Coriolis force. Along a vertical line in the figure $Ek$ increases with increasing $Pr$ (see also table 1). Thus viscous dissipation, which suppresses the TP effect, increases (relative to the Coriolis force) as $Pr$ increases at constant $Ra$ and $1/Ro$, thereby diminishing the $Nu$ enhancement by the TP effect.

A second important topic is the influence of rotation on the LSC. The LSC has several properties with interesting dependences on $1/Ro$ that we investigated and briefly describe below.

In the absence of rotation, plumes emitted by the top and bottom thermal BLs are swept by the LSC and travel vertically mostly near the sidewall. There they generate a destabilizing vertical thermal gradient that is much larger than the very small gradient along the axis of the system (Brown & Ahlers 2007b). We found that these gradients initially were diminished as $1/Ro$ increased from zero. They reached a minimum somewhat below $1/Ro_c$, followed by an increase until a strong enhancement was reached at large $1/Ro$ (see figures 10 and 11). There the enhancement was larger for larger $Pr$ at constant $Ra$, and larger for larger $Ra$ at constant $Pr$; compared to its value at $1/Ro = 0$, it reached a factor of 2.5 for some parameters. The strong enhancement presumably was associated with contributions from the Ekman-vortex structures; but the depression of the gradient below $1/Ro_c$ remains to be explained.

In the absence of rotation, the Reynolds number $Re$ of the LSC has often been determined from the frequency of oscillations detected by various experimental
techniques (Ahlers et al. 2009). We determined $Re$ from oscillations in auto-correlation functions of sidewall temperature measurements (see figure 12b) and found that $Re$ was initially unchanged within our resolution as $1/Ro$ increased, up to about $1/Ro = 0.3$ (see figure 12c). With further increase of $1/Ro$, it decreased, but we were unable to make measurements for $1/Ro \gtrsim 0.5$ because enhanced fluctuations of the system prevented the resolution of the correlation-function oscillations. The constant values of $Re$ stand in contrast to the strong decrease of the thermal gradient along the wall and to the strong increase of the LSC temperature amplitudes $\langle \delta_k \rangle$ to be discussed below.

An important issue is whether the LSC continues to exist as $1/Ro$ increases, or whether the evolving Ekman-vortex structure and associated vigorous temperature and velocity fluctuations lead to the complete destruction of the LSC as was concluded by some previous investigators (Hart et al. 2002; Kunnen et al. 2008b). From the analyses of time series of sidewall temperature measurements, we found that the LSC continued to exist up to $1/Ro \simeq 1$, but for larger $1/Ro$ the temperature fluctuations became too severe and the LSC amplitudes became too small for us to be sure of the LSC existence.

Remarkably, the time-averaged thermal amplitude $\langle \delta_k \rangle$ first increased with increasing $1/Ro$, but for $1/Ro \gtrsim 0.3$ it decreased and for $1/Ro \gtrsim 0.6$ it settled down at a near-constant value which was still about 60 or 70% of its value in the absence of rotation (see figure 13b). Interestingly, there was no obvious singularity in $\langle \delta_k \rangle$ at $1/Ro_c$. However, the LSC became obscured by fluctuations that grew by a factor of 2 or so as $1/Ro$ grew from 0 to greater than 1.

It is well known that for the non-rotating system the LSC undergoes rare cessations of its flow when $\langle \delta_k \rangle$ vanishes (Brown et al. 2005a; Brown & Ahlers 2006b). We found that the frequency of cessations was dramatically enhanced by even modest rotation (see figures 14 and 16). The cessation frequency increased from about $10^{-5}$ s$^{-1}$ for $1/Ro = 0$ to about $1.5 \times 10^{-4}$ s$^{-1}$ for $1/Ro = 0.2$. At $1/Ro_c$ there was a jump by a factor of about 3 in the cessation rate. These are interesting experimental observations, but like many other features of this system, they remain unexplained.

Finally, an intriguing aspect of this system is the retrograde rotation rate $\omega$ of the orientation of the LSC circulation plane. In the absence of dissipation, one would expect $\omega = -\Omega$ because the system has angular momentum and thus an orientation that, in the absence of an applied torque, should remain steady in an inertial frame. Small deviations from this, with $-\omega/\Omega \approx 0.8$, are expected due to viscous drag on the sample walls (Hart et al. 2002; Brown & Ahlers 2006a, 2008a). However, typical measurements yielded $-\omega/\Omega \lesssim 0.2$ (see figure 19). One would be tempted to invoke a turbulent viscosity to explain this larger drag, as was done by Hart et al. (2002). However, there is intricate structure of $\omega/\Omega$ as a function of $1/Ro$ in the range $1/Ro \lesssim 0.7$ or so that would then imply a similar structure of this turbulent viscosity, which would need to be explained.

All of the properties displayed in figures 11–19 show significant and intriguing structure at small $1/Ro$. In order to see whether there is any correlation between particular features of different properties, we show all of them for $Ra = 9.0 \times 10^6$ and $Pr = 4.38$ in figure 21 on the same horizontal scale. There the vertical dotted line is $1/Ro_c$ as determined from the Nusselt measurements. In figure 21(a), we also demonstrate the method of determining $1/Ro_c$ by the two solid lines which are fits to data subsets below and above the transition. The vertical short dashed line passes through the maximum of the LSC thermal amplitude $\langle \delta_m \rangle$ shown in figure 21(d). This point coincides with the value of $1/Ro$, where the slope of the vertical temperature
Figure 21. (a) The excess Nusselt number $\frac{Nu(1/Ro)}{Nu(0)}$; (b) the vertical thermal gradient $\frac{\Delta T_w(1/Ro)}{\Delta T_w(0)}$ near the sidewall; (c) the reduced Reynolds number $\frac{Re(1/Ro)}{Re(0)}$; (d) the thermal LSC amplitude $\langle \delta_m \rangle$; and (e) the retrograde rotation rate $-\omega/\Omega$ of the LSC. All are shown on the same horizontal scale as a function of $1/Ro$ and are for $Pr = 4.38$ and $Ra = 9.0 \times 10^9$.

The gradient $\frac{\Delta T_w(Ro)}{\Delta T_w(0)}$ (figure 21b) changes more or less abruptly. However, the Reynolds number shown in figure 21(c) begins to drop earlier and shows no remarkable behaviour at that point. The minimum of the retrograde rotation rate, shown in figure 21(e), occurs slightly below and not at the vertical dashed line. The apparent correlation between $\frac{\Delta T_w(Ro)}{\Delta T_w(0)}$ and $\langle \delta_m \rangle$ may indeed be significant because one would expect a more vigorous LSC to lead to lesser vertical temperature...
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gradients; but it is surprising that neither of these properties shows any anomaly at $1/\text{Ro}_c$. It is difficult to see any significant correlation between the behaviour of $\text{Re}(\Omega)/\text{Re}(0)$ or $-\omega/\Omega$ and the other properties, but again it is noteworthy that these last two also pass smoothly through $1/\text{Ro}_c$. It seems that the presumed onset of Ekman-vortex formation at $1/\text{Ro}_c$ does not have a large effect on the various properties of the LSC.

There still remain numerous issues that are inadequately studied by experiment or DNS. Among them is the absence of a bifurcation as a function of $1/\text{Ro}$ for $\text{Ra} \simeq 4 \times 10^7$ (Stevens et al. 2010a), while such a bifurcation exists for $\text{Ra} \gtrsim 10^8$ (Stevens et al. 2009). One would like to know whether this transition occurs by virtue of a vanishing slope $S_1(\text{Ra})$ of $\text{Nu}(1/\text{Ro})$ at a critical value $\text{Ra}^*$ of $\text{Ra}$. In view of the data in figure 7, this seems unlikely, and in any case, it would require an explanation of the existence of yet another bifurcation, this time as a function of $\text{Ra}$. Instead, one might like to know whether the bifurcation as a function of $1/\text{Ro}$ disappears discontinuously, with finite $S_1$ above and $S_1 = 0$ below a transition at some $\text{Ra}^*$ in the range $4 \times 10^7 \lesssim \text{Ra}^* \lesssim 10^8$. This too would correspond to a bifurcation along the $\text{Ra}$ axis, and presumably would be related to a discontinuous change in the nature of the turbulent state of RBC without rotation as $\text{Ra}$ drops below about $10^8$ or so (see e.g. Heslot, Castaing & Libchaber 1987). Alternatively, one might ask whether $S_1$ simply becomes sufficiently small with decreasing $\text{Ra}$ to be no longer resolved by the DNS. This question could in principle be answered by DNS or experiments at closely spaced values of $\text{Ra}$ that would extend the data for $S_1$ in figure 7 to smaller values of $\text{Ra}$. Similarly, one would like to know in more detail the parameter ranges at small $\text{Pr}$ or large $\text{Ra}$ over which the Ekman enhancement disappears for any $\text{Ro}$ (Zhong et al. 2009; Niemela et al. 2010).

Even more issues are awaiting a theoretical explanation. Prominent among these are the $1/\text{Ro}$-dependences of several properties at relatively small $1/\text{Ro} \lesssim 0.5$ or so. These properties include the Nusselt number, the vertical thermal gradient near the sidewall, the LSC Reynolds number, the LSC thermal amplitude $\langle \delta_k \rangle$, the size of the temperature fluctuations about the azimuthal sinusoidal temperature profile along the sidewall, the frequency of cessations of the LSC and the retrograde rotation rate of the LSC. Equally important would be an explanation of the very existence of a bifurcation, rather than a continuous evolution towards the Ekman state. A quantitative treatment of the size of the Ekman enhancement and of the TP depression of the heat transport likewise seems to be lacking at this time. It is apparent that much remains to be done on this problem.

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