The influence of rotation on turbulent Rayleigh-Bénard convection

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Convection in a cylindrical sample viewed from above. Ra = 8x10^8, Pr = 29.

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Polar low over the Barents Sea on February 27, 1987
Fluid properties

\[ \text{Ra} = \left( \frac{\alpha}{\kappa} \nu \right) g L^3 \Delta T \]

\[ g = 9.8 \text{ m} / \text{s}^2 \]

Large-scale circulation (LSC)

Aspect ratio \( \Gamma = \frac{D}{L} = 1.0 \) and 0.5

Nusselt number \( \text{Nu} = \frac{Q L}{\Delta T} / \lambda \)

Prandtl number \( \text{Pr} = \frac{\nu}{\kappa} \) 3 to 6.4

See e.g. Ahlers, Grossmann, and Lohse, Rev.Mod.Phys. 81, 503 (2009).
Without rotation

1.) High-precision heat-transport measurements have been made.

2) The LSC has been studied extensively.

\[ T_{i,0} = \langle T \rangle + \delta_0 \cos(i\pi/4 + \theta_0), \quad i = 0, \ldots, 7 \]
Over long time periods: 
Net rotation.
Over shorter times:

Azimuthal diffusion

\[
\dot{\theta}_n \equiv \frac{\theta_0(t + n\delta t) - \theta_0(t)}{n\delta t}
\]

\[
\langle \theta_0(t + n\delta t) - \theta_0(t) \rangle_{rms} = \sqrt{D_\theta n\delta t}
\]

Ra = 5x10^9
The effect of deliberate rotation about a vertical axis


Two new effects:

I. Ekman pumping at modest rotation rates
II. Taylor-Proudman suppression at larger rotation rates
Under modest rotation, 
Ékman vortices extract hot (cold) fluid from the bottom (top) boundary layer.

Taylor-Proudman: Under strong rotation vertical heat transport is inhibited because
\[
\frac{\partial v}{\partial z} \approx 0
\]
There is a supercritical bifurcation from one turbulent state to another!

Ekman vortices form only for $1/\text{Ro}_c = 0.38$?

Red solid circles: Expt. Open squares: DNS
Ra = 1.2x10^9
Pr = 3.05 (squares)
Pr = 4.38 (diamonds)
Pr = 6.32 (triangles)

Pr = 4.38
Ra = 1.8x10^{10} (triangles)
Ra = 8.9x10^9 (squares)
Ra = 2.2x10^9 (solid circ.)
Ra = 1.2x10^9 (diamonds)
Ra = 5.6x10^8 (open circ.)
Pr = 4.38

Black solid circ: $\Gamma = 1$ $Ra = 2.2 \times 10^9$

Pink circ: $\Gamma = 0.5$ $Ra = 4.5 \times 10^9$

Purp. squares: $\Gamma = 0.5$ $Ra = 9.0 \times 10^9$

Blue Diamonds $\Gamma = 0.5$ $Ra = 1.8 \times 10^{10}$

$\Gamma = 0.5$: $1/Ro_c = 0.86$

$\Gamma = 1.0$: $1/Ro_c = 0.41$

$1 / Ro_c \sim 1 / \Gamma$

Finite size effect.
No bifurcation in the infinite system.
Finite size effect. No bifurcation in the infinite system.

Γ = 0.5

Γ = 1.0

Γ = 2.0

Ra = 2.8x10^8

1/Ro = 0.05

1/Ro = 0.6
We shall make a model for the vortex density $A$

$Ra = 2.8 \times 10^8$

$1/Ro = 0.67$
$1.00$
$1.54$
$3.33$

$1/Ro \sim 3$
Ginzburg-Landau equation

\[ \dot{A} = \frac{1}{\text{Ro}^2} A - gA^3 + \xi_0^2 \nabla^2 A \]

For the infinite uniform system:

\[ A = g^{-1/2}(1/\text{Ro}) \propto \Omega \]
\[ A = 0 \]

\( \nabla^2 A \) is the lowest-order allowed gradient term

Stability of \( A=0 \): substitute \( A = A_0 \exp(\sigma t) \exp(ikx) \) into linear equation:

\[ \sigma = \frac{1}{\text{Ro}^2} - \xi_0^2 k^2 \]

Neutral curve (\( \sigma = 0 \)):

\[ \frac{1}{\text{Ro}_0^2} = \xi_0^2 k^2 \]

Boundary conditions:

\[ A(-\Gamma/2) = A(\Gamma/2) = 0 \]

\[ k_0 = \frac{\pi}{\Gamma} \]

\[
A(x) = (Ro^2 g)^{-1/2} \tanh((0.5 - x)/\xi)
\]
\[
\xi = \sqrt{2}\xi_0 Ro \simeq 0.07
\]

\[
\bar{A} = \bar{\bar{g}}^{-1/2} \left( \frac{1}{Ro} \right) \left( \frac{1}{Ro_c} \right)
\]

\[
\frac{Nu(1/\text{Ro})}{Nu(0)} - 1 = S_1(\Gamma) \left( \frac{1}{Ro} \right) \left( \frac{1}{Ro_c} \right)
\]
Summary:

1.) Ekman pumping occurs only above \(1/Ro_c \sim 1/\Gamma\) where, at modest \(1/Ro\), it enhances the heat transport.

2.) \(1/Ro_c \sim 1/\Gamma\) can be understood as a finite-size effect which can be described by a Ginzburg-Landau-like model.

3.) The infinite system thus would have no bifurcation.