Experiments on Wave Number Selection in Rotating Couette-Taylor Flow

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Experimental results are reported for the selection of axial wavelengths of Taylor vortices in a system where the Reynolds number, \( R \), is “ramped” spatially from above to below the critical value, \( R_c \), for the onset of vortex flow. It is found that a sufficiently slow ramp connecting the supercritical region (containing vortices) to a subcritical region (of purely azimuthal flow) results in the selection of a unique wavelength. A more rapid ramp results in a small \( R \)-dependent band of allowed wavelengths which grows in width as \( R \rightarrow R_c \) from above.

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In nonlinear dissipative systems subjected to an external stress, \( R \), a transition frequently occurs to a state of reduced symmetry having a spatial structure with a characteristic wavelength. Examples are Rayleigh–Bénard convection, Taylor vortex flow, certain chemical reactions, flame-front propagation, and crystal growth. The equations governing systems of infinite spatial extent usually possess a continuum of linearly stable solutions, corresponding to a band of wave numbers having a width which varies as \( \varepsilon^{1/2} (\varepsilon = R/R_c - 1) \), and \( R_c \) is the value of the stress parameter at the transition). Imposition of nonperiodic boundary conditions may reduce the width of the allowed band to order \( \varepsilon \), but for the investigated cases a finite band remains. Such finite systems generally select discrete states from the band of stable solutions, with the selected wave numbers dependent upon the particular boundary conditions imposed.

For Taylor vortex flow (TVF) between concentric rotating cylinders, the multiplicity of stable states was first explored in detail by Coles and Snyder. They observed TVF to be stable for a range of axial wavelengths when the Reynolds number, \( R \), was above \( R_c \), the value for vortex formation, but below the value where azimuthal waves appear. The observable range of wavelengths is roughly equal to that expected theoretically, but the factors determining the wavelength selected in a particular experiment are not understood. It has been suggested quite recently, however, that a gradual axial variation of \( \varepsilon \) from positive to negative values (a spatial ramp) could permit a continuous wavelength adjustment mechanism to become effective, which might result in the selection of a unique although possibly \( \varepsilon \)-dependent state. The question of the existence and nature of such selection processes is largely unexplored. One might expect that their nature would have a strong influence on the development of temporally nonperiodic (or turbulent) behavior in large real systems. More specifically, we would expect a “weak” selection mechanism to render the primary modes of the system relatively sensitive to the influence of noise and other degrees of freedom of the system.

To investigate wavelength selection experimentally, we used apparatus in which the gap between the cylinders tapered linearly from supercritical to subcritical values as shown in Fig. 1. The working fluid was a 30% solution of glycerol in water by volume with 0.6% by volume of Kalliroscope suspension added for flow visualization.

FIG. 1. Schematic diagram of the apparatus (not to scale).
The mean, steady-state, axial wavelength $\bar{\lambda}$ of the 6–7 vortex pairs in the supercritical straight section was measured with a cathetometer (excluding the three vortices nearest the collar). By movement of the collar (see Fig. 1) in small steps, $\bar{\lambda}$ was measured as a function of aspect ratio $L$ (straight section length/gap). Figure 2 shows the results of such measurements with $\alpha = 0.016$, for several values of $\epsilon$ (in the straight section). Note that positive and negative steps in $L$ gave the same results within our resolution. Clearly, a range of stable wavelengths exists in this case, and the available range decreases rapidly with increasing $\epsilon$. For a given $L$, a unique wavelength is selected out of the range, and this wavelength has a period of $\sim 2$ in $L$. Wavelength adjustment occurred continuously by movement of vortices through the ramp. To search for other possible stable flows, states with wavelengths both larger and smaller than those of Fig. 2, by as much as $15\%$, were created by various protocols. Such states were always observed to decay to the state expected on the basis of Fig. 2.

Similar experiments were also carried out by use of the ramp with $\alpha = 0.002$. For $\epsilon > 0.01$, no evidence of a stable band was obtained in the moving-boundary experiments. Instead, $\bar{\lambda}$ remained essentially fixed at 2.0 as $L$ was varied. This result was confirmed by decay experiments at larger values of $L$. The latter measurements revealed a very slight decrease ($\sim 0.02$) in the selected $\bar{\lambda}$ as $\epsilon$ was increased to 0.1. Data for $\epsilon = 0.002$ show that a noticeable band does develop even for $\alpha = 0.002$, provided $\epsilon$ is sufficiently small.

Figure 3 summarizes the present findings. It also shows results obtained in similar experiments using apparatus with the same cylinder radii but containing only a straight section $\approx 20$ cm long terminated by nonrotating collars. Under the latter conditions, wavelength adjustment is discontinuous and involves creation of or annihilation of a vortex pair somewhere in the interior of the system. The figure also shows the theoretically predicted range over which, in the small-$\epsilon$ limit, TVF is stable to infinitesimal axisymmetric perturbations.

We may compare our experimental results with the properties of two different model systems of equations. The first is the amplitude equation:

$$\tau_0 \ddot{A} = \xi_0 A'^2 A + A(\epsilon - |A|^2),$$

(1)

where the dot and double prime denote first time and second spatial derivatives, respectively. In

FIG. 2. Mean axial wavelength of steady-state Taylor vortices as a function of aspect ratio $L$ for several $\epsilon$ and a ramp angle of 0.016 rad. The solid circles were obtained by increasing and the open circles by decreasing the aspect ratio in small steps.

FIG. 3. Range of stable wavelengths for TVF as a function of $R/R_c$. Solid line: boundary of linearly stable TVF (Ref. 13). Solid circles: experiments in the absence of a ramp ($\alpha = 1/2$) (Ref. 12). Horizontal bars: range for $\alpha = 0.016$. Crosses: values for $\alpha = 0.002$. 

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Eq. (1), $A$ is a complex amplitude containing only slow spatial variations, and the full axial dependence of the velocity components is given by the stream function $\psi = \text{Re}[A \exp(i k_c z)]$, where $k_c$ is the wave number at $R = R_c$. The parameters $\tau_0 = 0.0381$ and $\xi_0 = 0.2654$ have been calculated from the linearized Navier-Stokes equation for our cylinder radii. For sufficiently small $\epsilon$, and for spatial variations which are slow compared to $\xi_0$, Eq. (1) should provide an accurate description of the flow. We solved it numerically in the presence of various ramps and find that a unique state with $\lambda = 2\pi/k_c$ is always selected, even for a step from $\epsilon > 0$ to $\epsilon < 0$. Thus, Eq. (1) does not contain the physics needed to describe the data of Fig. 2.

The second set of model equations solved numerically are the reaction-diffusion equations

\[ \begin{align*}
\tau_1 \dot{A}_1 &= D_1 A_1'' + a_1 A_1 (1 - A_1^2) - b_1 A_2, \\
\tau_2 \dot{A}_2 &= D_2 A_2'' - a_2 A_2 (1 - A_2^2) + b_2 A_1,
\end{align*} \tag{2} \]

for $A_1$ and $A_2$ real, and with the parameters $D_1 = 0.03758$, $D_2 = 0.1747$, $a_1 = 1$, $a_2 = 1.2$, and $b_1 = b_2 = b_0 [1 - \epsilon(\xi)]$. They have stable periodic solutions for $b_1$, $b_2$ less than a critical value. Their only connection with the actual fluid flow is that they reduce to Eq. (1) for small $\epsilon$. As in Ref. 15, we found a band of stable states for nonzero $\alpha$ which increased in width roughly in proportion to $\epsilon$, contrary to the behavior of the real system. However, a previously unreported feature of the solutions is that the boundaries and the spatial ramp select discrete states within the band. For small $\epsilon$, there is only one state for a given aspect ratio, and its wavelength is a periodic function of $L$ with period 1 as shown in Fig. 4. This behavior is very similar to the results shown in Fig. 2 for the real system; however, there the period is about 2. Evidently for the Couette-Taylor system, inflow and outflow boundaries are not equivalent in their interaction with the spatial ramp, whereas for the equations all half-cycles behave identically. The above analysis suggests that expansion of the equations of motion for TVF to next-higher order beyond the amplitude equation is worthwhile. Such an expansion might well contain the new features of the data, i.e., the existence for finite $\alpha$ of a band whose width increases with decreasing $\alpha$ and the period-2 dependence upon $L$ of the wavelength of the selected state.

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8. See, for instance, Ref. 1.
9. M. C. Cross, P. G. Daniels, P. C. Hohenberg, and

FIG. 4. Wavelength of steady-state solutions to the reaction-diffusion equations [Eq. (2)] as a function of aspect ratio $L$ for several $\epsilon$. We used $\alpha = 0.10$. 

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E. D. Siggia, Phys. Rev. Lett. 45, 898 (1980), and to be published.


M. A. Domínguez-Lerma, G. Ahlers, and D. S. Cannell, to be published. We scale lengths by the gap $d$ and time by $d^2/\nu$ ($\nu$ is the kinematic viscosity).